Why sensitivity analysis
(or: How not to do a sensitivity analysis)

Andrea Saltelli

Sensitivity Analysis Summer School, Parma, June 2024

upf. BARCELONA SCHOOL OF MANAGEMENT

UNIVERSITAS BERGENSIS

The Politics of Modelling

Numbers Between Science and Policy

Andrea Saltelli and Monica Di Fiore

"*The Politics of Modelling: Numbers between Science and Policy* is a breath of fresh air and a much-needed cautionary view of the ever-increasing dependence on mathematical modelling in ever-widening directions. The five aspects of modelling that should be 'minded' are a sensitive summary of factors that should be considered when evaluating any mathematical model."

ORRIN H. PILKEY, PROFESSOR, DUKE UNIVERSITY'S NICHOLAS SCHOOL OF THE ENVIRONMENT, CO-AUTHOR, WITH LINDA PILKEY-JARVIS, OF *USELESS ARITHMETIC: WHY ENVIRONMENTAL SCIENTISTS CAN'T PREDICT THE FUTURE*, COLUMBIA UNIVERSITY PRESS, WASHINGTON, DC, 2009

Where to find this talk: www.andreasaltelli.eu

Andrea Saltelli

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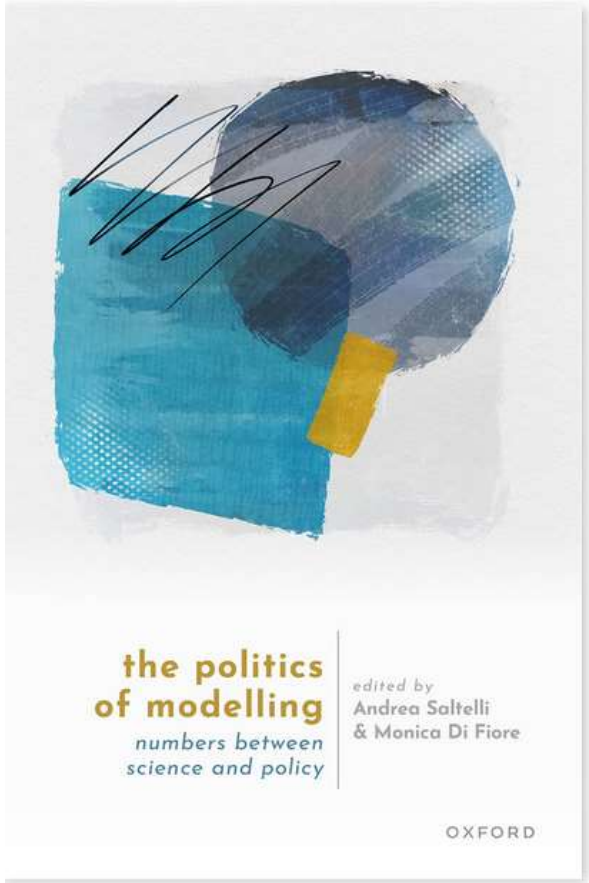
August 25 2023: The politics of modelling is out!



Praise for the volume

"A long awaited examination of the role —and obligation —of modeling."
Nassim Nicholas Taleb , Distinguished Professor of Risk Engineering, NYU Tandon School of Engineering. Author, of the 5 -volume series Incerto.

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"A breath of fresh air and a much needed cautionary view of the ever-widening dependence on mathematical modeling."
Orrin H. Pilkey, Professor at Duke University's Nicholas School of the Environment, co-author with Linda Pilkey-Jarvis of Useless Arithmetic: Why Environmental Scientists Can't Predict the Future, Columbia University Press 2009.

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"The methods by which power insinuates itself into models, and facilitates their portability and

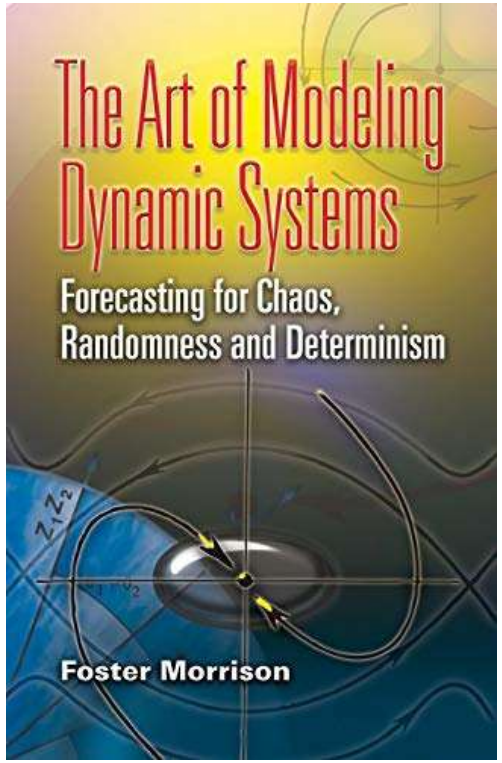
Mastodon Toots by @AndreaSaltelli

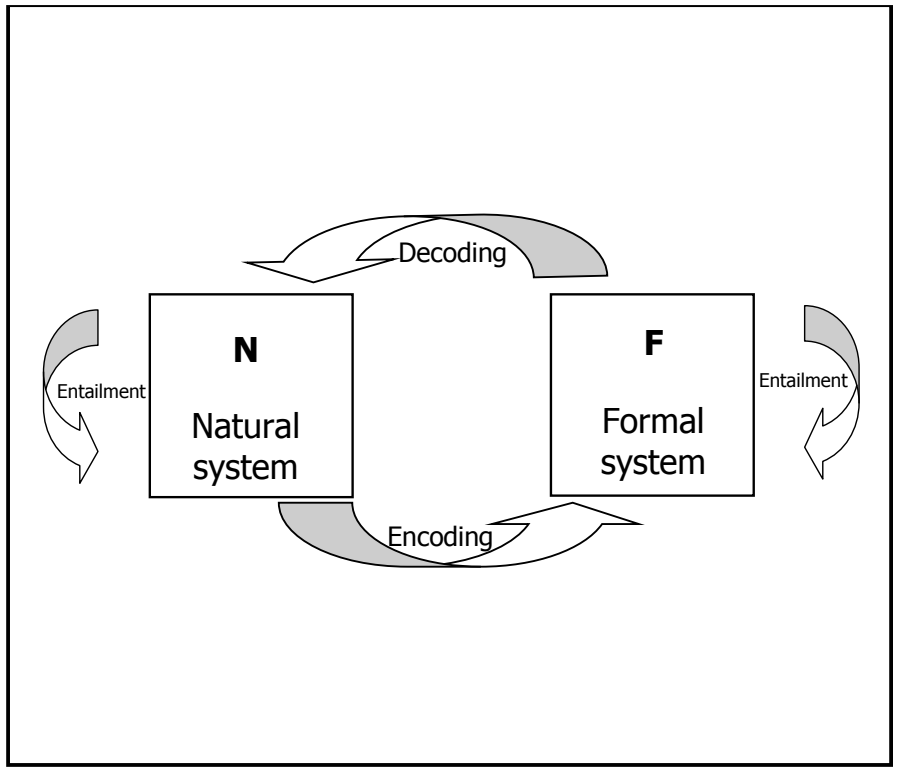
AndreaSaltelli
2024/2/16 16:38
Are you interested in sensitivity analysis? Here a short story of it.
Open access, online in draft form: sciencedirect.com/science/arti

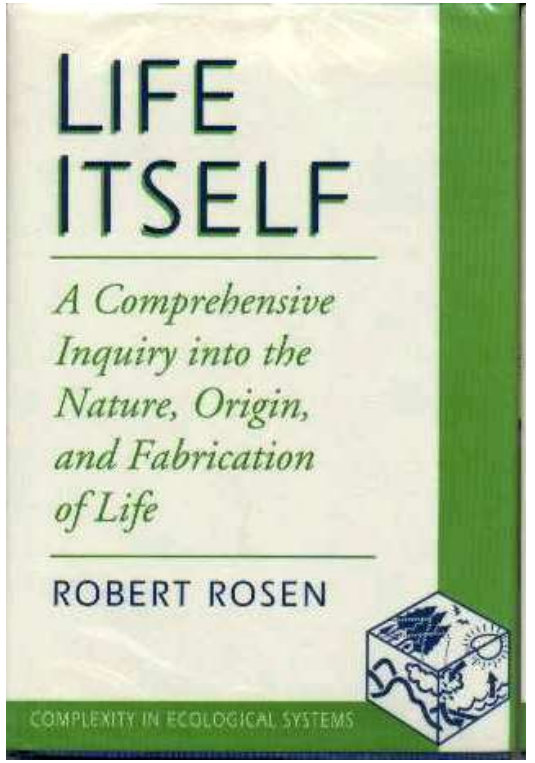
Andrea

View on mstdn.social

Modelling is a craft (or art) more than a science







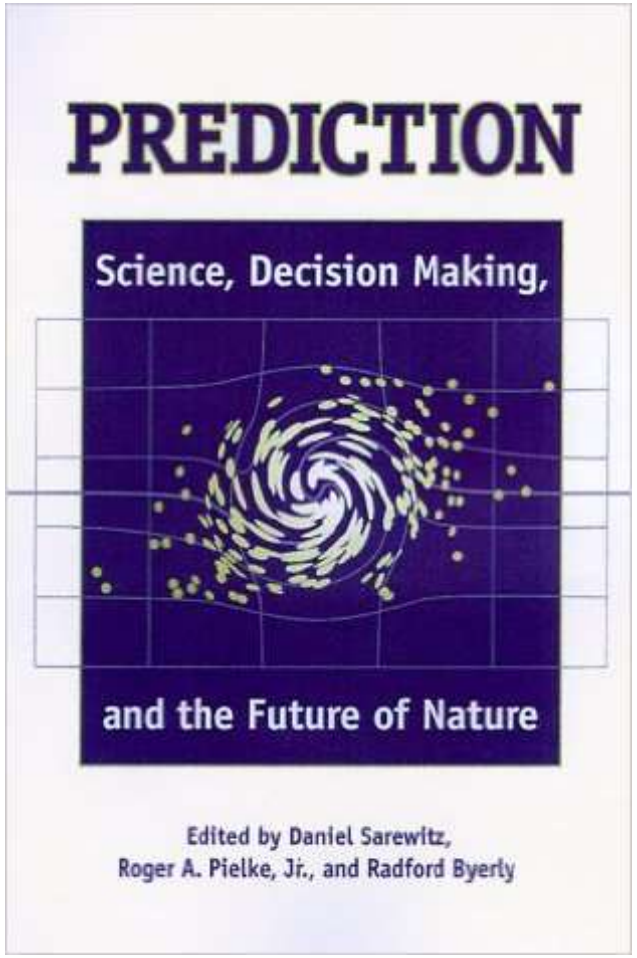
Louie, A.H. 2010. "Robert Rosen's Anticipatory Systems." Foresight 12 (3): 18–29.
Padilla, J. J., Diallo, S. Y., Lynch, C. J., & Gore, R. (2018). Observations on the practice and profession of modeling and simulation: A survey approach. SIMULATION, 94(6), 493–506.

“models are most useful when they are used to challenge existing formulations, rather than to validate or verify them”

Naomi Oreskes

N. Oreskes, K. Shrader-Frechette, and K. Belitz, “Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences,” Science, 263, no. 5147, 1994.

Models are not physical laws



Oreskes, N., 2000, Why predict? Historical perspectives on prediction in Earth Science, in Prediction, Science, Decision Making and the future of Nature, Sarewitz et al., Eds., Island Press, Washington DC

“When a model generates a prediction, of what precisely is the prediction a test? The laws? The input data? The conceptualization?

Any part (or several parts) of the model might be in error, and there is no simple way to determine which one it is”

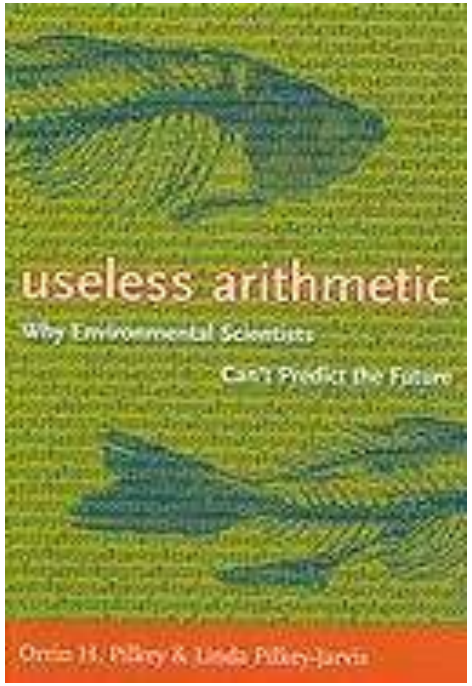
→ Duhem-Quine critique

Don’t confuse the map with the territory

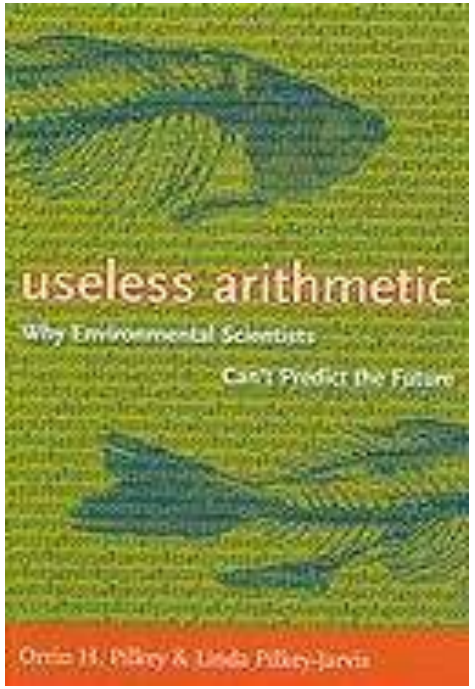
If you do, sensitivity analysis will not save you

<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature>>

Orrin H. Pilkey



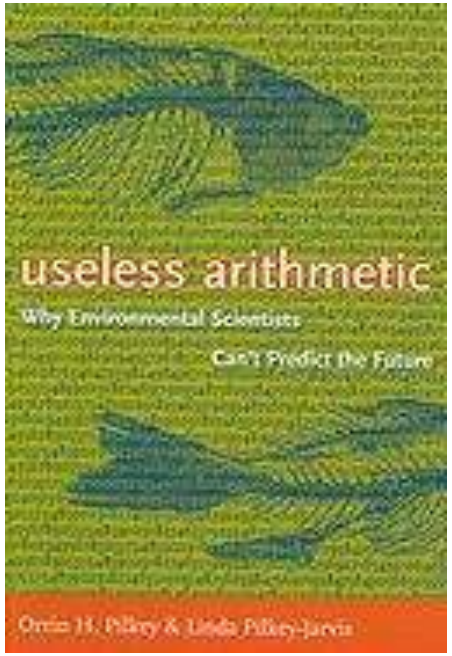
Useless Arithmetic: Why Environmental Scientists Can't Predict the Future
by Orrin H. Pilkey and Linda Pilkey-Jarvis, Columbia University Press, 2009.

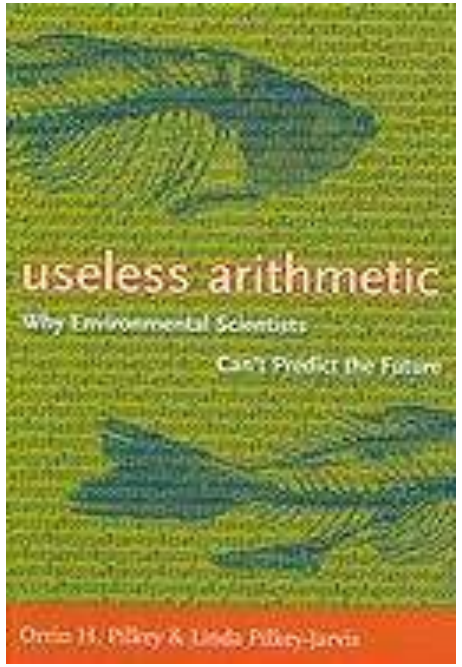


<<…If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit>>

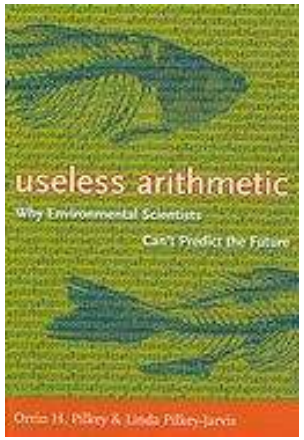
One of the examples discussed concerns the Yucca Mountain repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

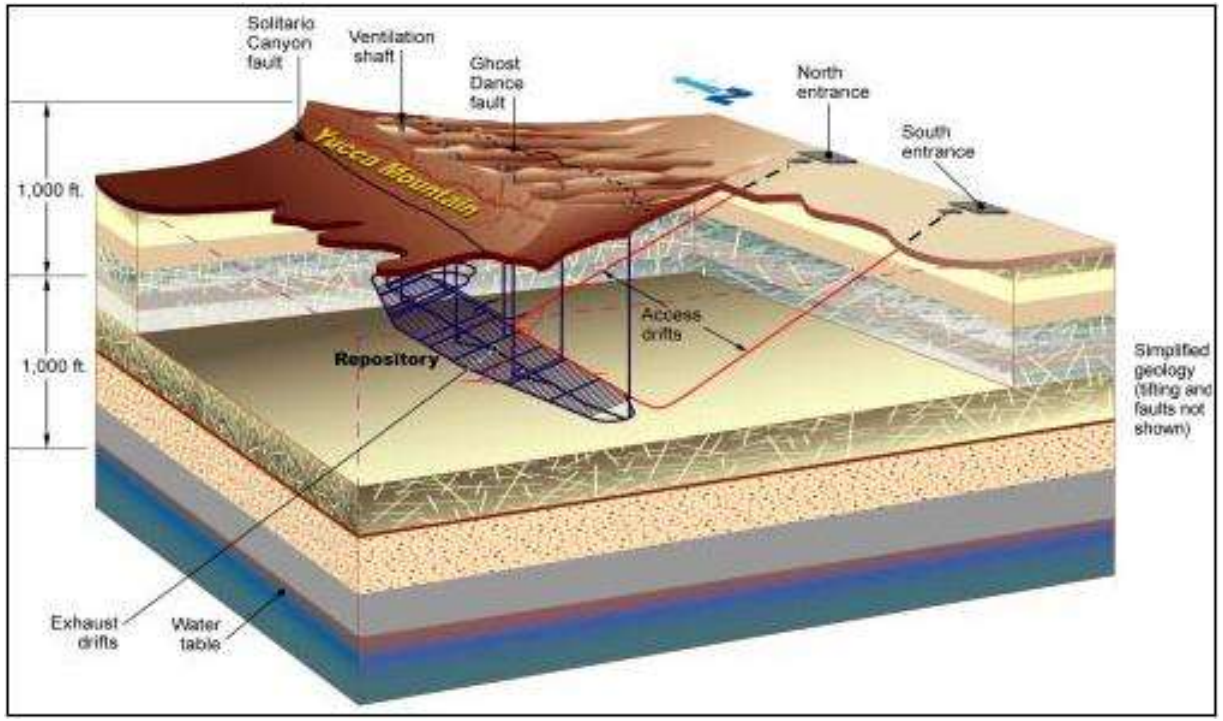
TSPA is Composed of 286 sub-models.





TSPA (like any other model) relies on assumptions → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal

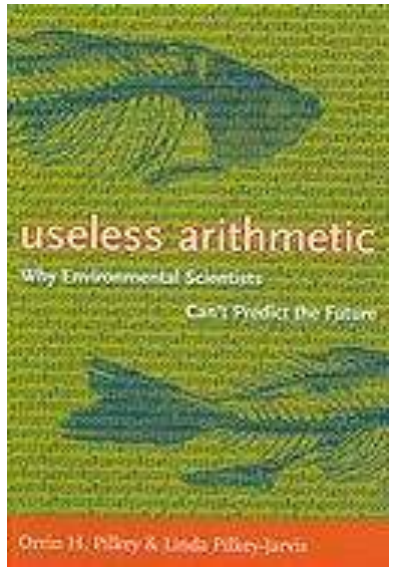




The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter
(the ^{36}Cl story)

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→… SA useless if it is instead ~ 3,000 millimetres per year.



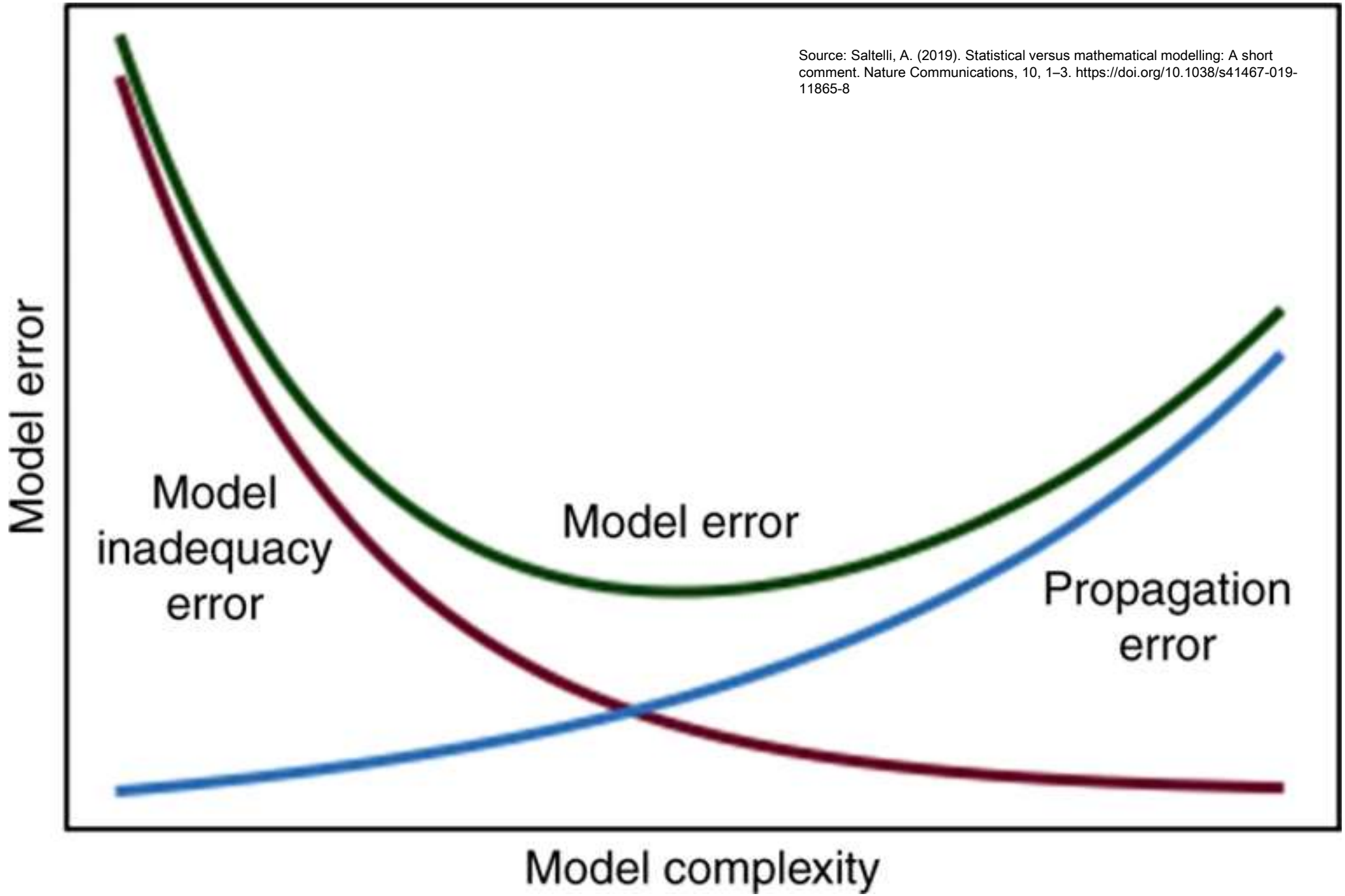
"Scientific mathematical modelling should involve constant efforts to falsify the model"

➔ Organized skepticism (as per CUDOS)

Communalism, Universalism, Disinterestedness, Organized Skepticism, from sociology of science, Robert K. Merton.

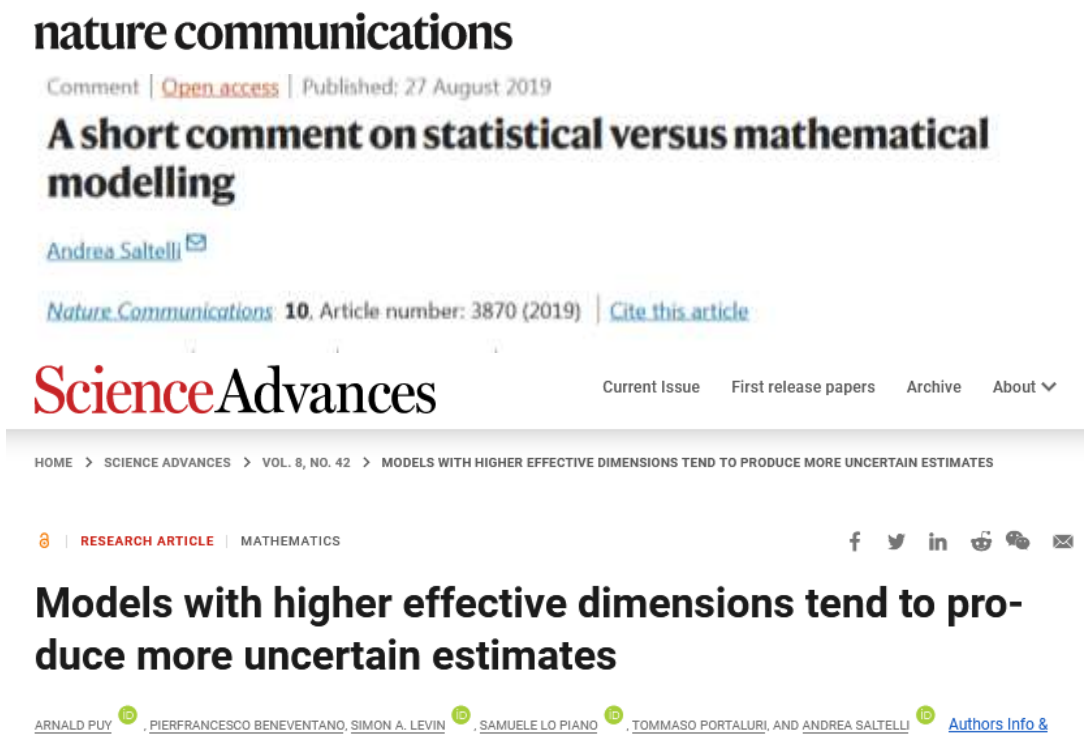
Beware the size of your model

Mind the conjecture of O'Neil



Source: Saltelli, A. (2019). Statistical versus mathematical modelling: A short comment. Nature Communications, 10, 1–3. https://doi.org/10.1038/s41467-019-11865-8

The conjecture of O'Neill (1971): too simple a model may miss important features of the system, and thus lead to systematic error, while a too complex one – high number of estimated parameters, may lead to a greater imprecision due the error propagation.



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A short comment on statistical versus mathematical modelling

Andrea Saltelli

Nature Communications **10**, Article number: 3870 (2019) | Cite this article

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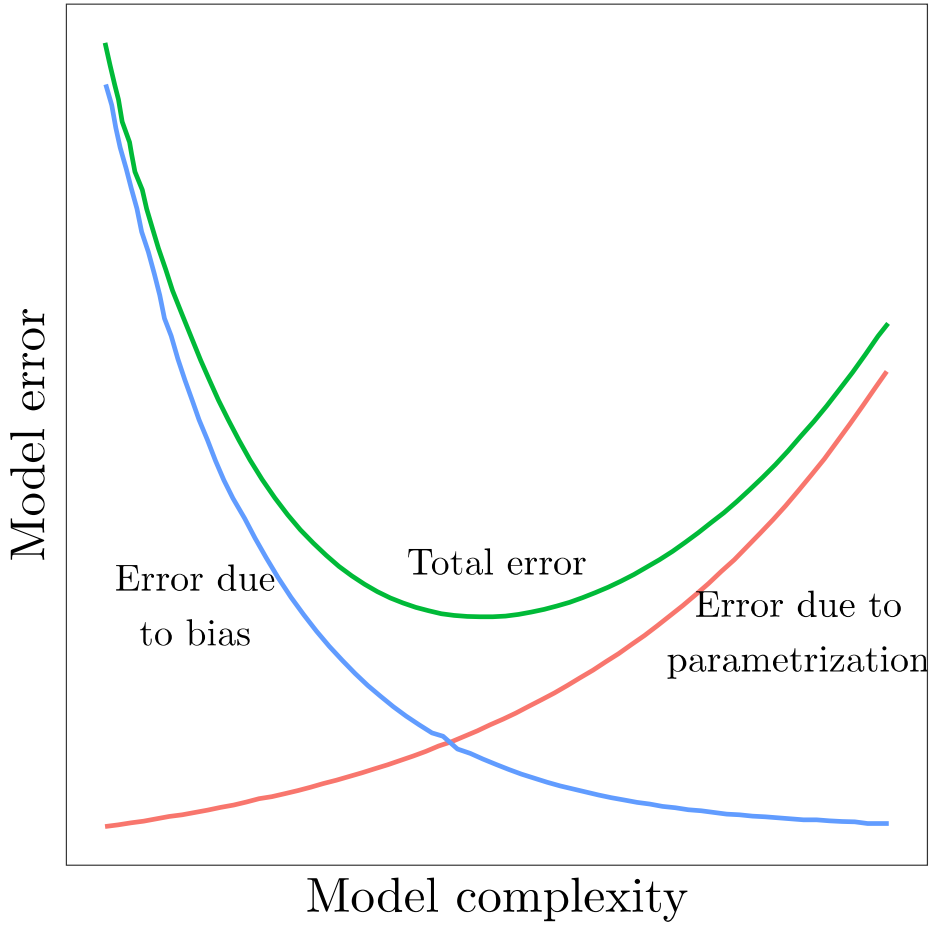
RESEARCH ARTICLE | MATHEMATICS

Models with higher effective dimensions tend to produce more uncertain estimates

ARNALD PUY, PIERFRANCESCO BENEVENTANO, SIMON A. LEVIN, SAMUELE LO PIANO, TOMMASO PORTALURI, AND ANDREA SALTELLI Authors Info &



What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

$$y = f(x_1, x_2, \dots x_k)$$

$$\left.\frac{\partial y}{\partial x_i}\right|_{x_i = x_i^0} \longleftarrow \text{Local}$$

What method would one choose to perform sensitivity analysis?

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$$y = f(x_1, x_2, \dots x_k)$$

$$\frac{x_i^0}{y^0}\frac{\partial y}{\partial x_i}\bigg|_{x_i = x_i^0} \longleftarrow \text{Local}$$

What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

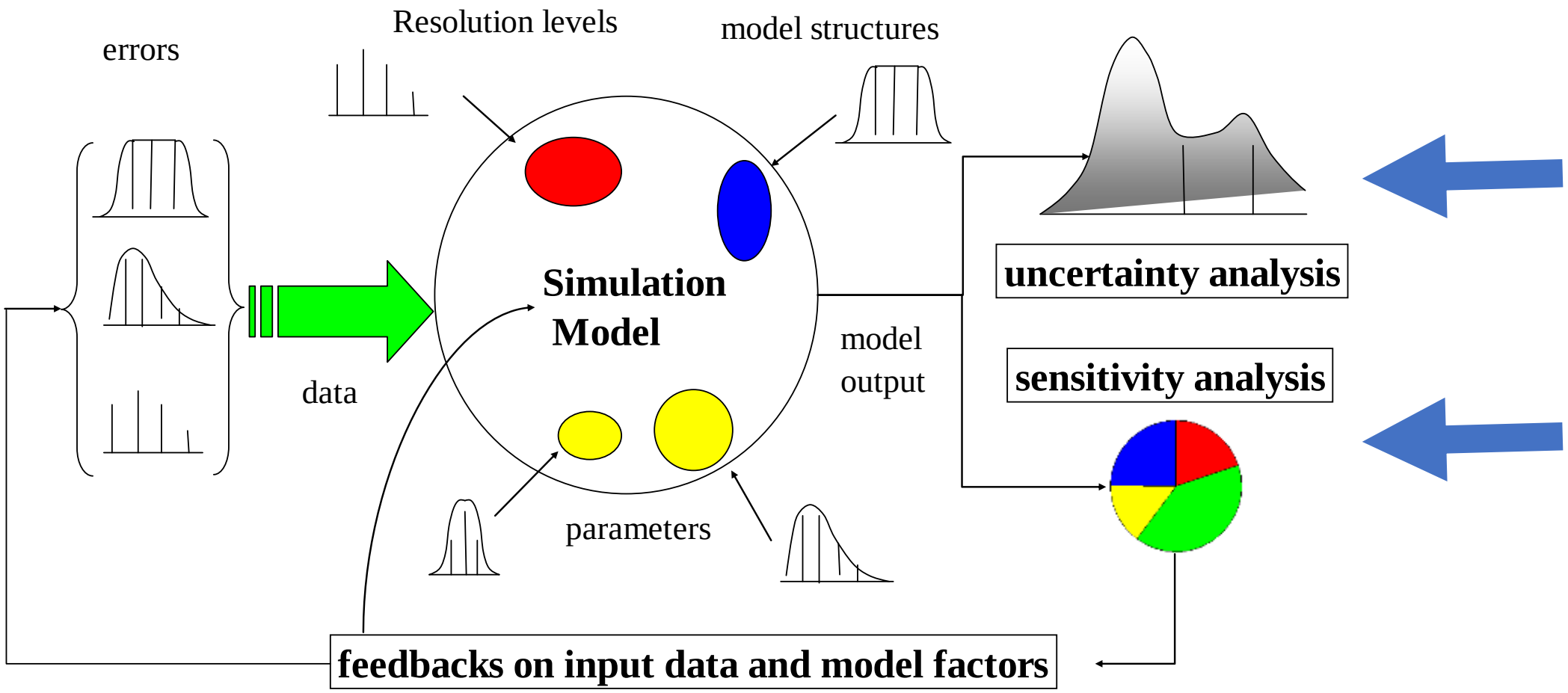
$$y = f(x_1, x_2, \dots x_k)$$

$$\left.\frac{std(x_i)}{std(y)}\frac{\partial y}{\partial x_i}\right|_{x_i = x_i^0} \longleftarrow \text{Hybrid}$$

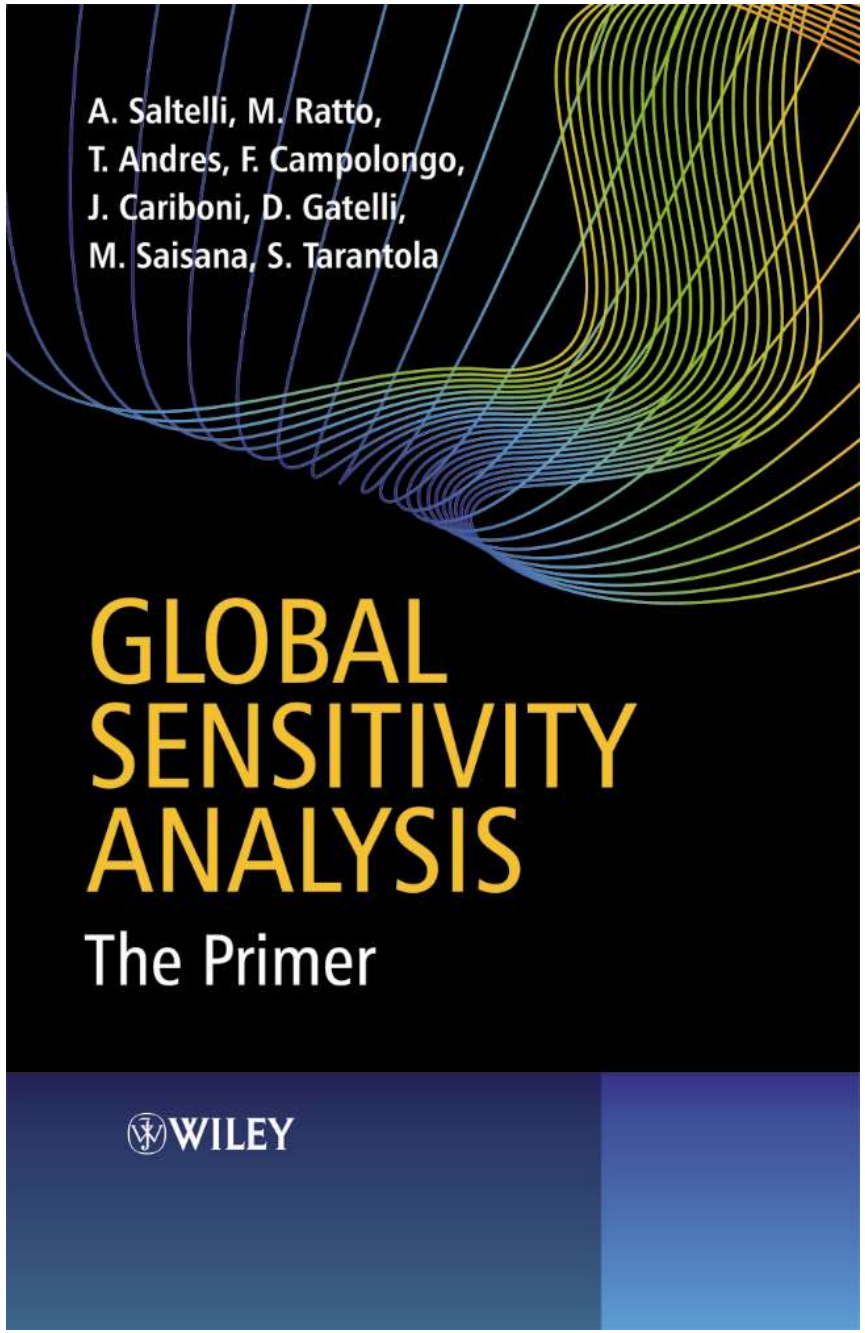
$$\left.\frac{\partial y}{\partial x_i}\right|_{x_i=x_i^0}$$ ← Relative effect on y of perturbing x_i around its nominal value

$$\left.\frac{x_i^0}{y^0}\frac{\partial y}{\partial x_i}\right|_{x_i=x_i^0}$$ ← Relative effect on y of perturbing x_i by a fixed fraction of its nominal value

$$\left.\frac{std(x_i)}{std(y)}\frac{\partial y}{\partial x_i}\right|_{x_i=x_i^0}$$ ← Relative effect on y of perturbing x_i by a fixed fraction of its standard deviation


errors
Resolution levels
model structures
Simulation Model
data
model output
parameters
uncertainty analysis
sensitivity analysis
feedbacks on input data and model factors

An introduction to variance based methods


A. Saltelli, M. Ratto,
T. Andres, F. Campolongo,
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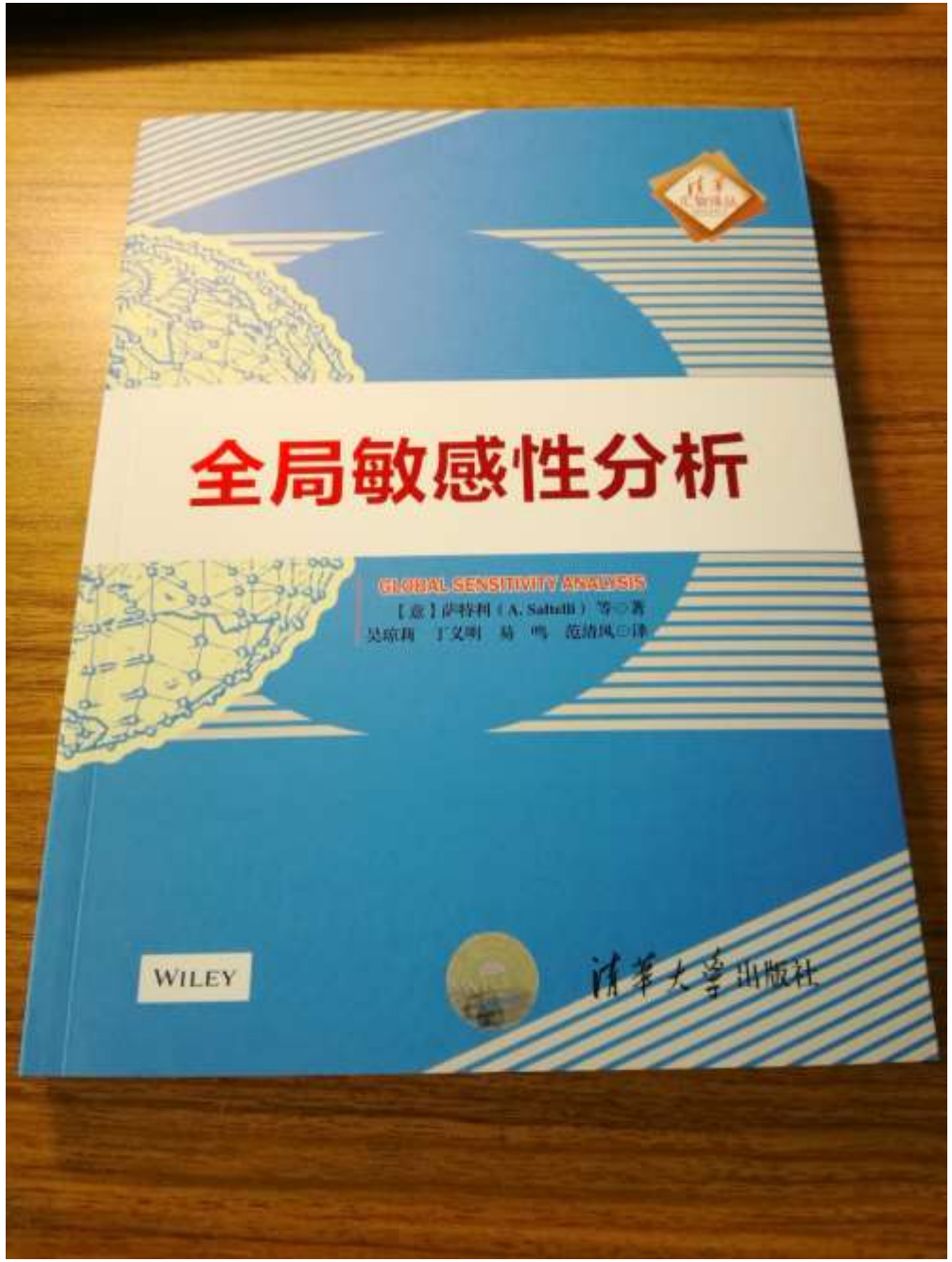
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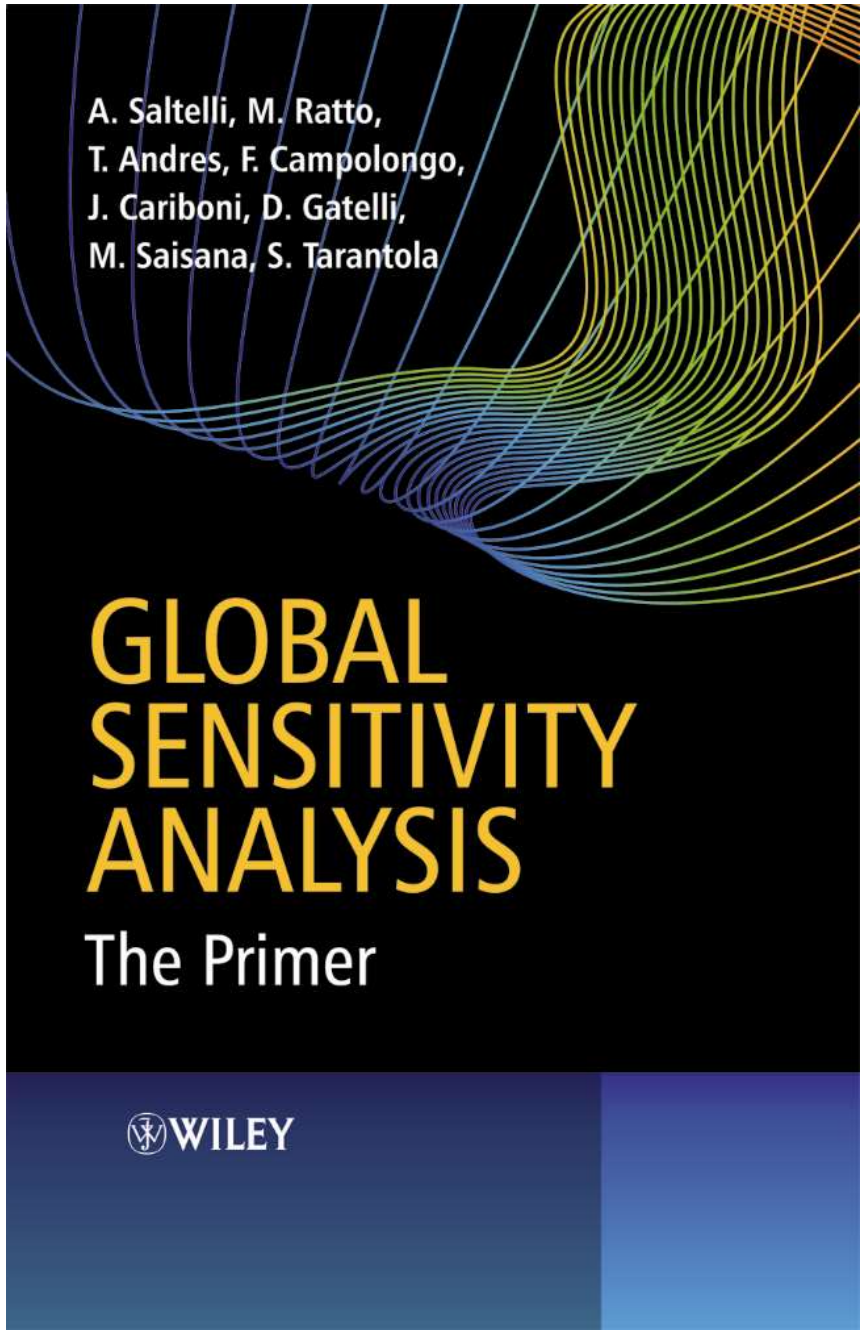

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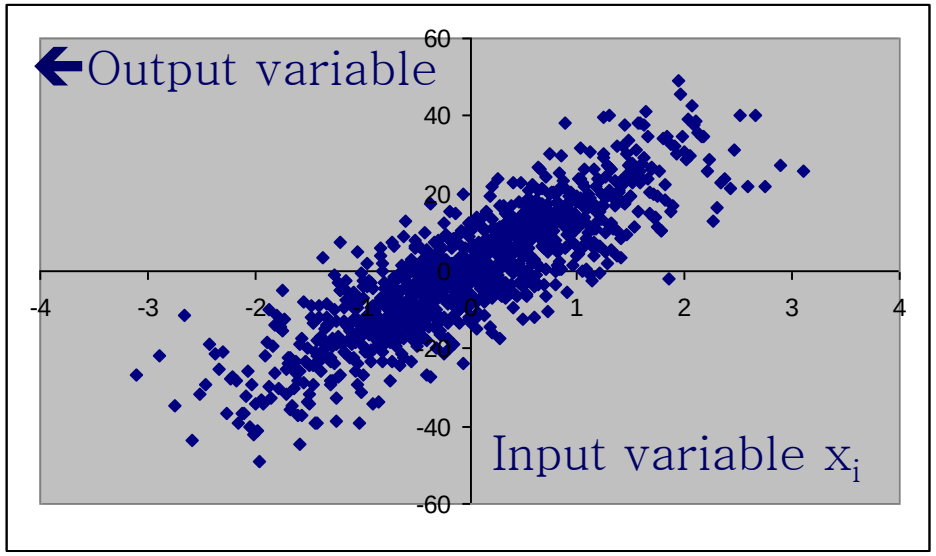
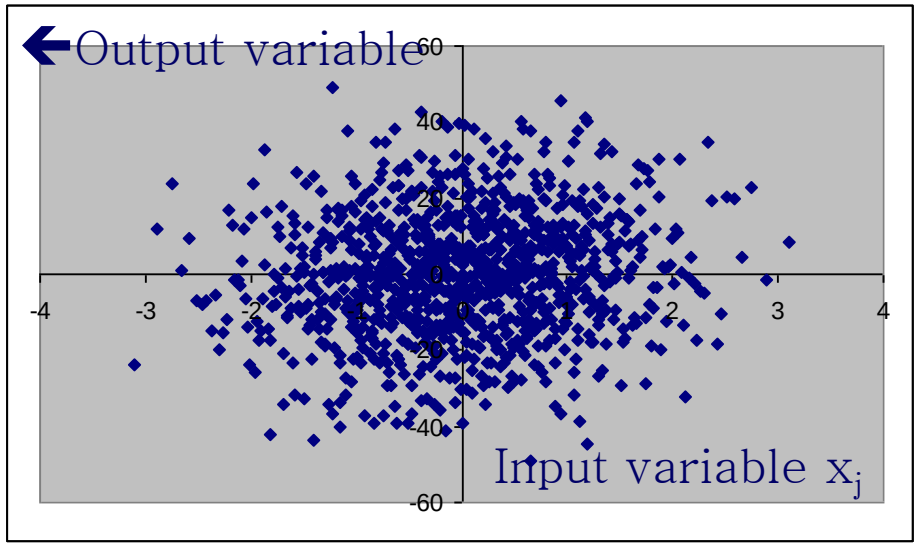
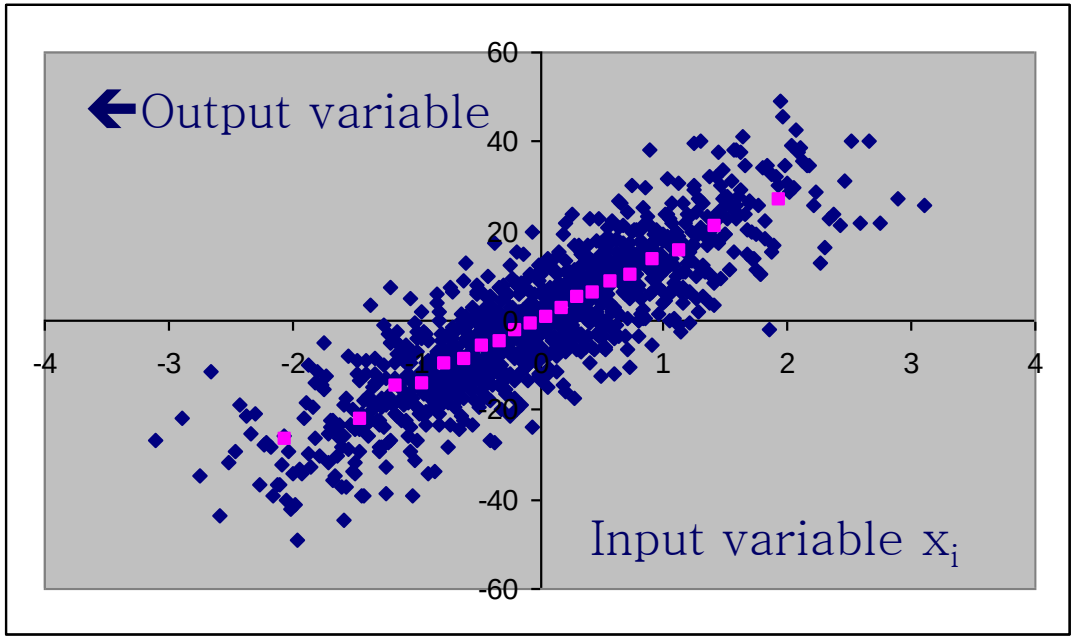
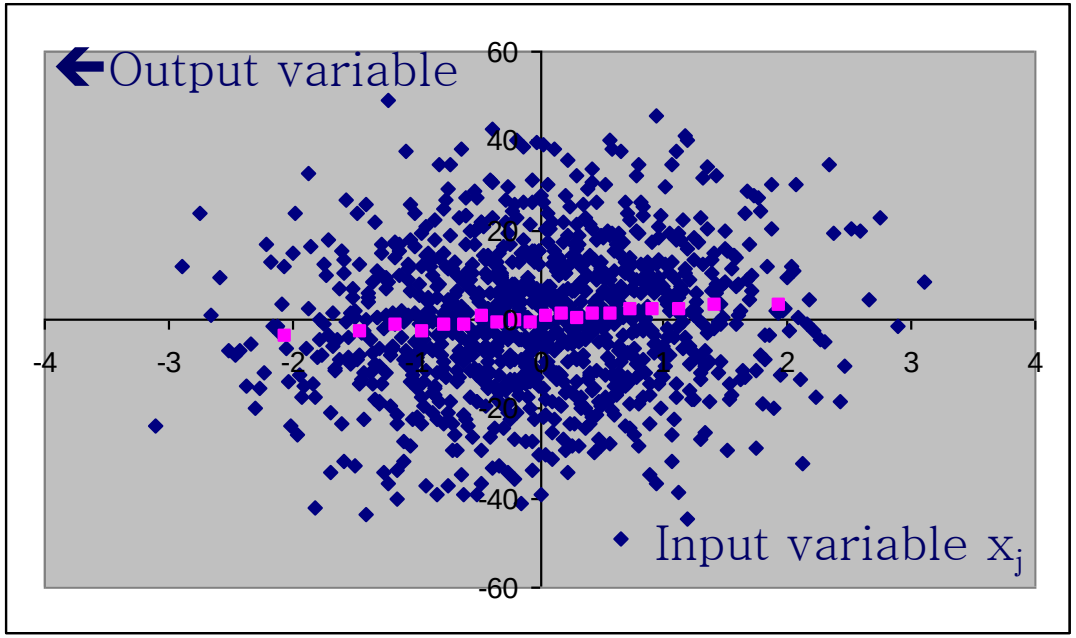




Plotting the output as a function of two different input factors

Which factor is more important?

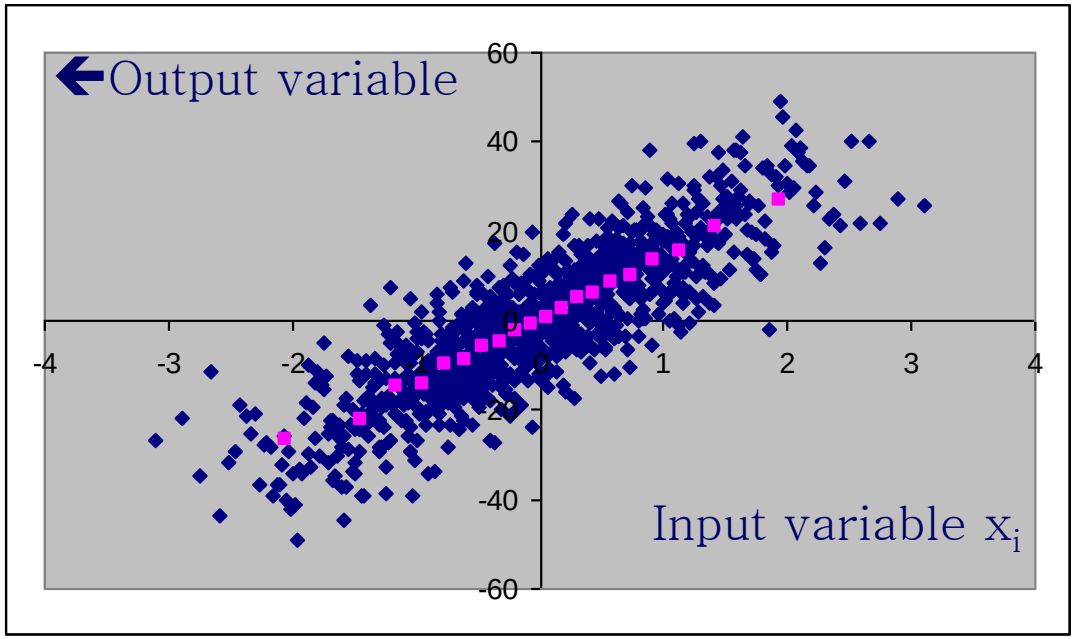




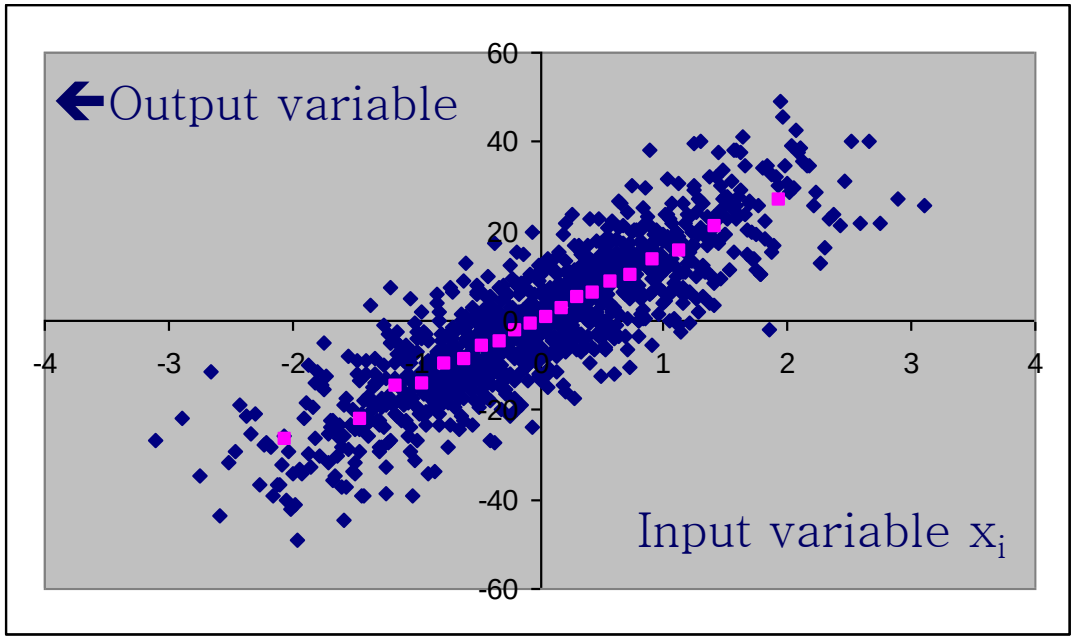
~1,000 blue points

Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)

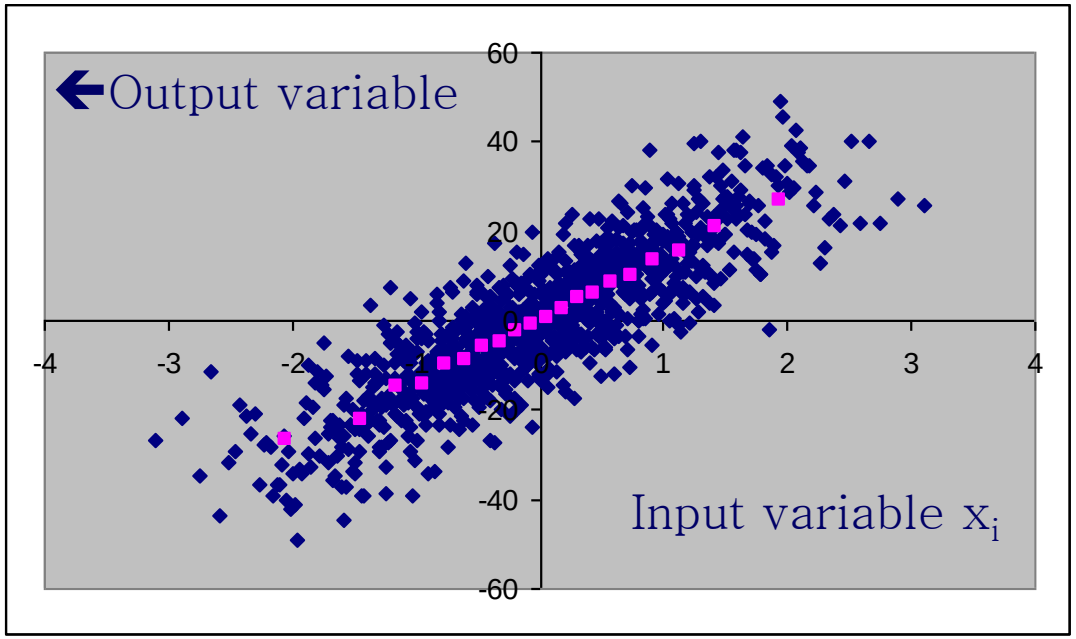


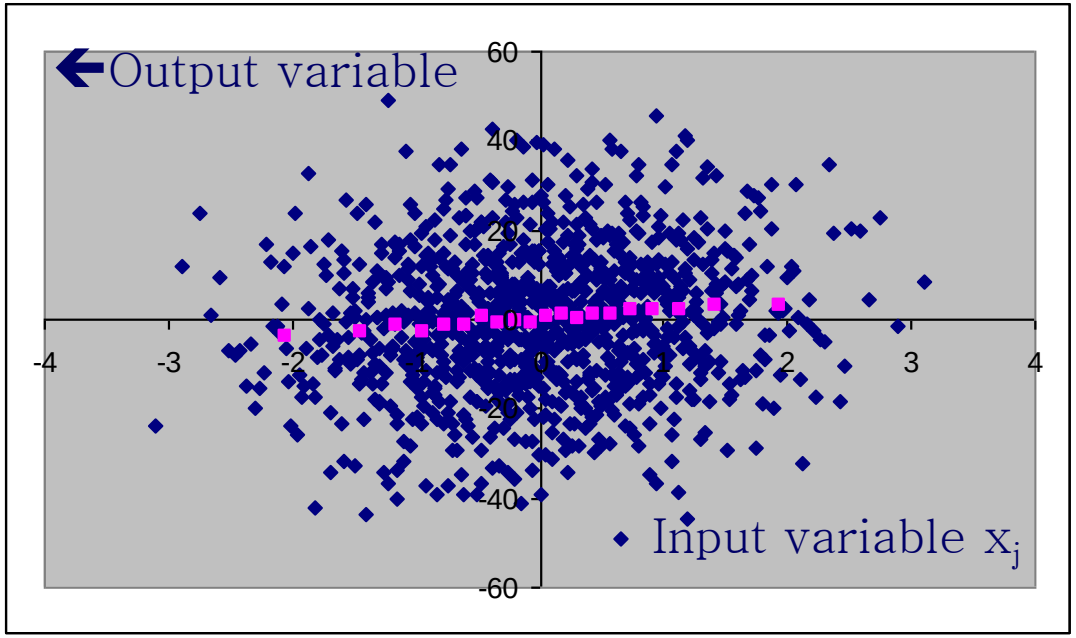
Each pink point is ~ $E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)$



Take the variance of the pink points one obtains a sensitivity measure

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$





Which factor has the highest

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)?$$

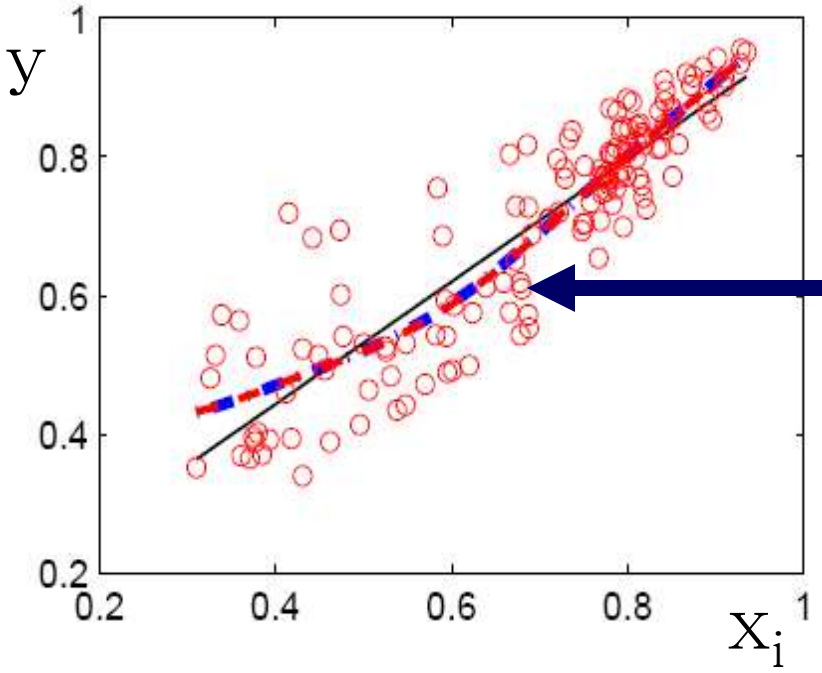
For additive models one can decompose the total variance as a sum of those partial variances

$$\sum_i V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right) \approx V(Y)$$

… which is also how additive models are defined

$$S_i = \frac{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)}{V(Y)}$$

The partial variance divided by the total variance is the so-called sensitivity index of the first order, identical in formulation to Pearson's correlation ratio



Smoothed curve:

$$\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)$$

First order sensitivity index:

$$\frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)}$$

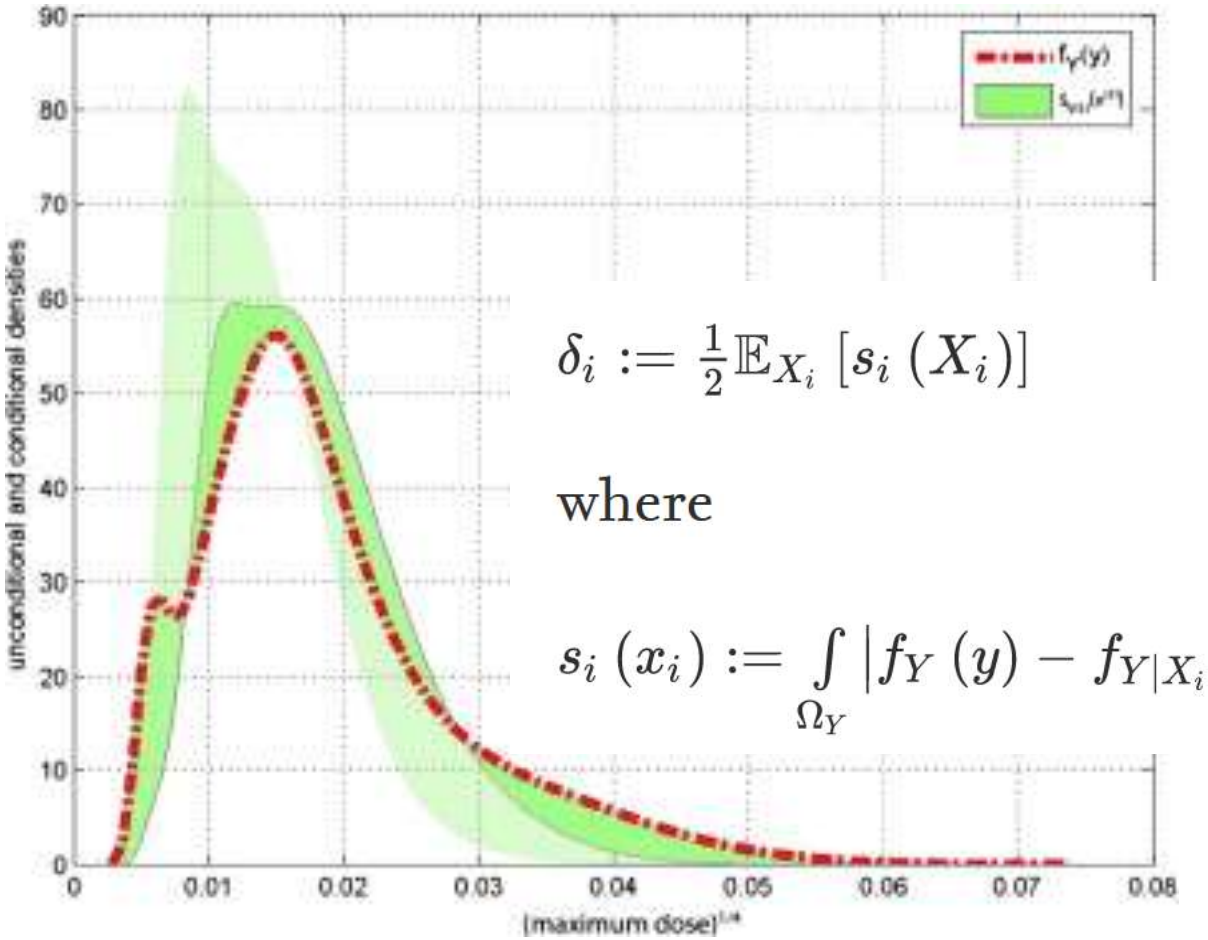
Advantages with variance based methods:

- graphic interpretation scatterplots
- statistical interpretation (ANOVA)
- expressed plain English
- working with sets
- relation to settings such as factor fixing and factor prioritization
- give the effective dimension

- Sergei Kucherenko tomorrow

Chapter 1 and its exercises

… but there are other methods that can be used for different settings, e.g. moment independents methods, Shapley coefficients, reduced spaces, VARS …



$$\delta_i := \frac{1}{2}\mathbb{E}_{X_i}\left[s_i\left(X_i\right)\right]$$

where

$$s_i\left(x_i\right) := \int_{\Omega_Y} \left|f_Y\left(y\right) - f_{Y|X_i=x_i}\left(y\right)\right| dy$$

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Environmental Modelling & Software

Volume 34, June 2012, Pages 105-115

Model emulation and moment-independent sensitivity analysis: An application to environmental modelling

E. Borgonovo [a], W. Castaings [b, c], S. Tarantola [d]

Don’t use One factor At a Time (OAT)

A geometric proof

Environmental Modelling & Software 25 (2010) 1508–1517

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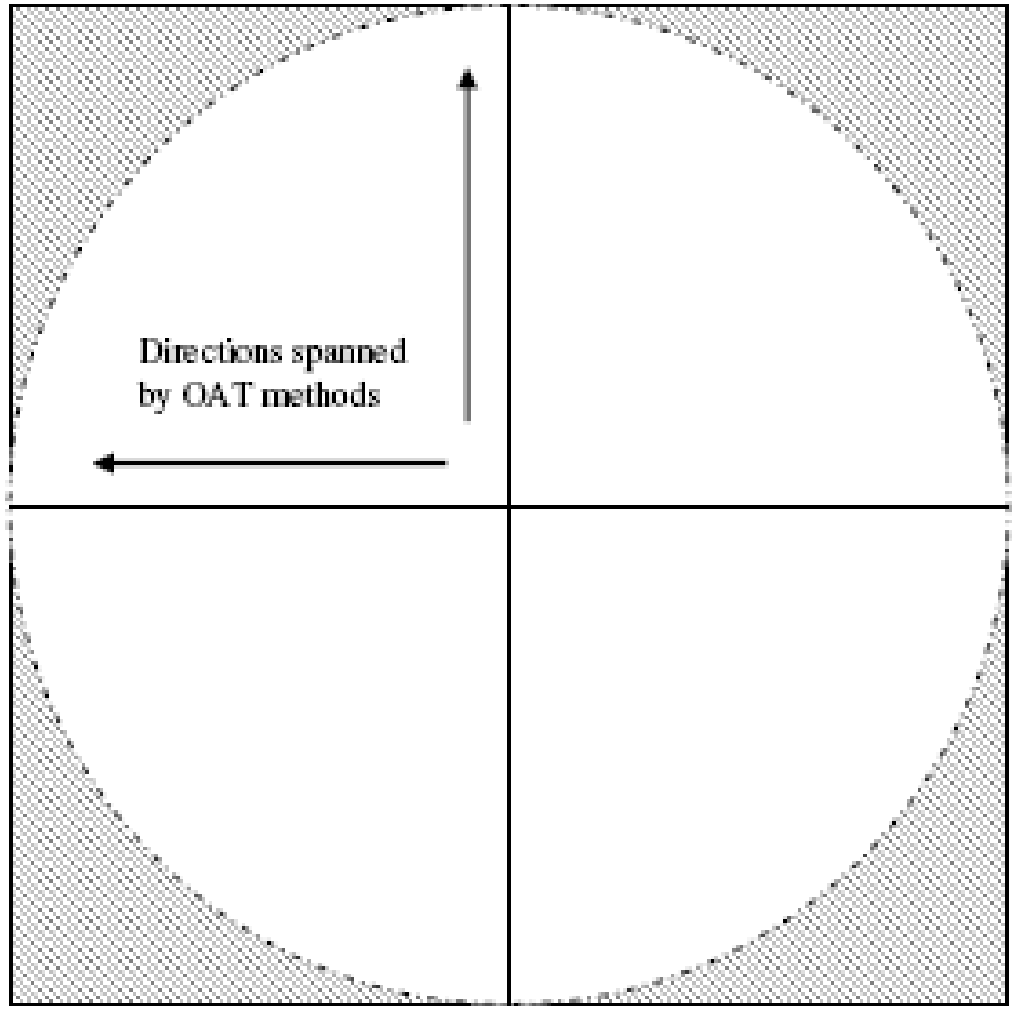
How to avoid a perfunctory sensitivity analysis

Andrea Saltelli*, Paola Annoni

Joint Research Center, Institute for the Protection and Security of the Citizen, via E.Fermi, 2749, Ispra VA 21027, Italy

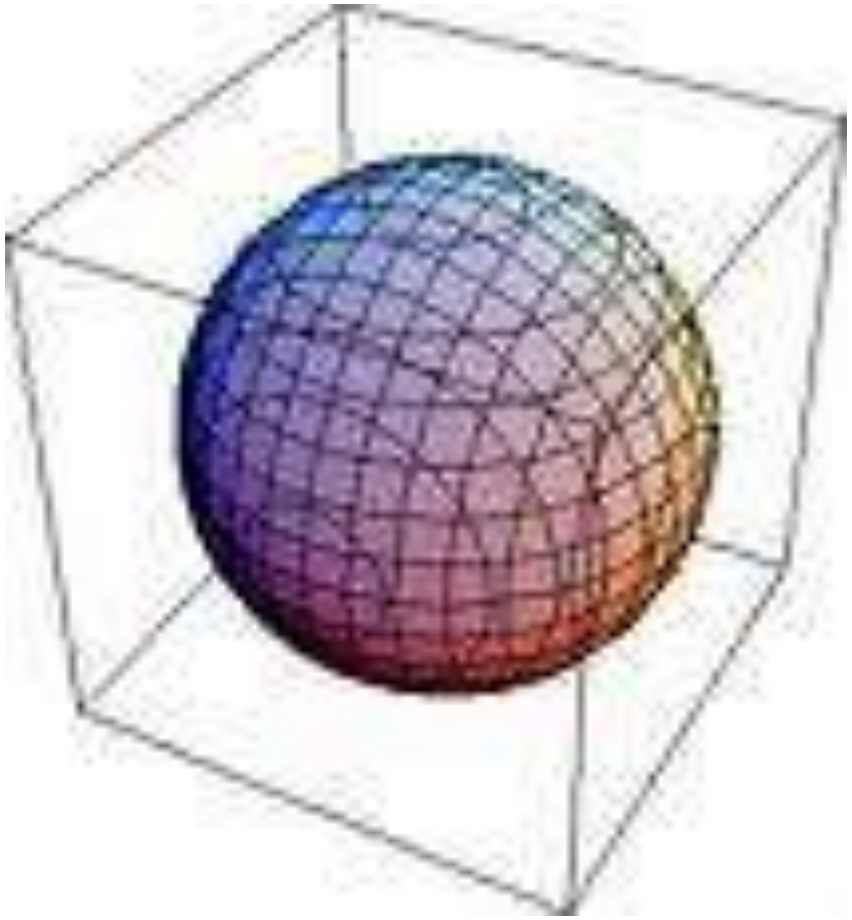
OAT in 2 dimensions



Area circle / area square =?

~ 3/4

OAT in 3 dimensions

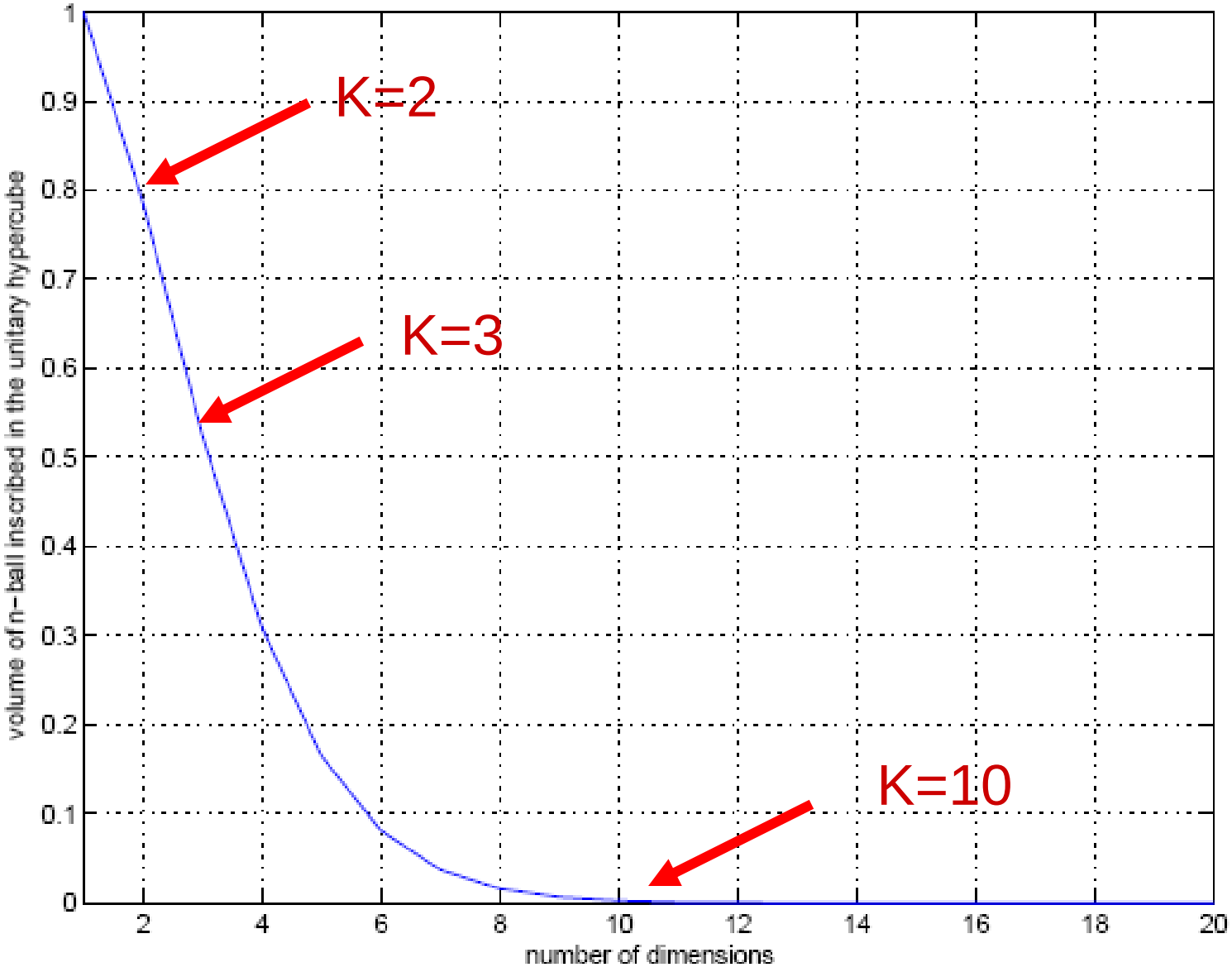


Volume sphere / volume cube =?

~ 1/2

OAT in 10 dimensions; Volume hypersphere / volume ten dimensional hypercube =? ~ 0.0025

OAT in k dimensions

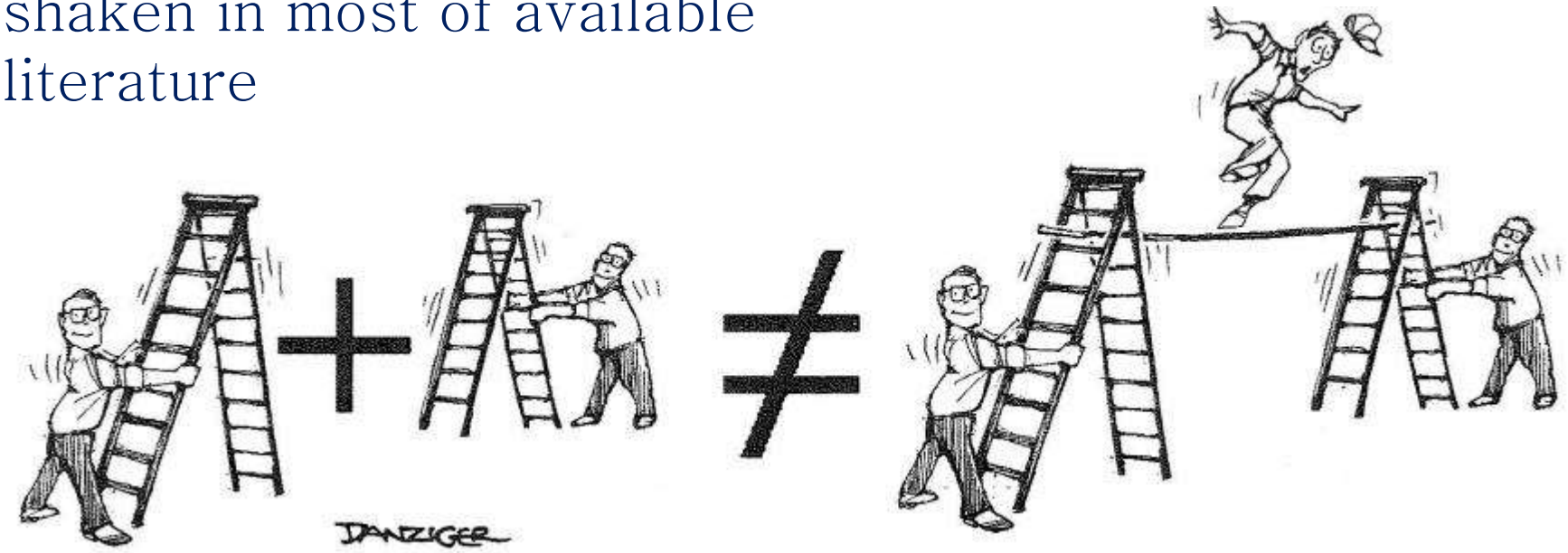


OAT does not capture interactions

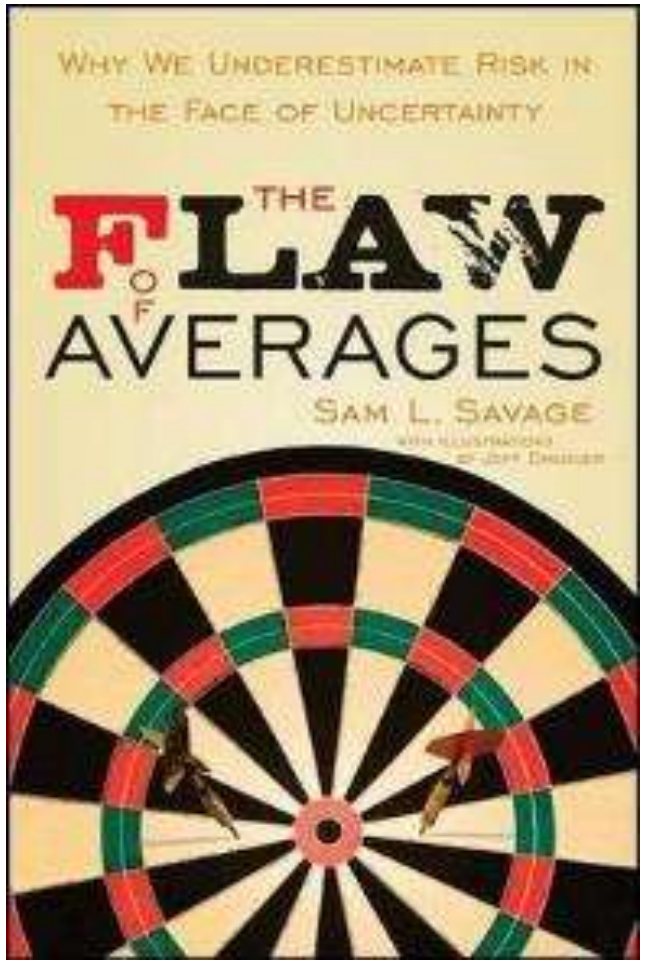
➔ The resulting analysis is non conservative

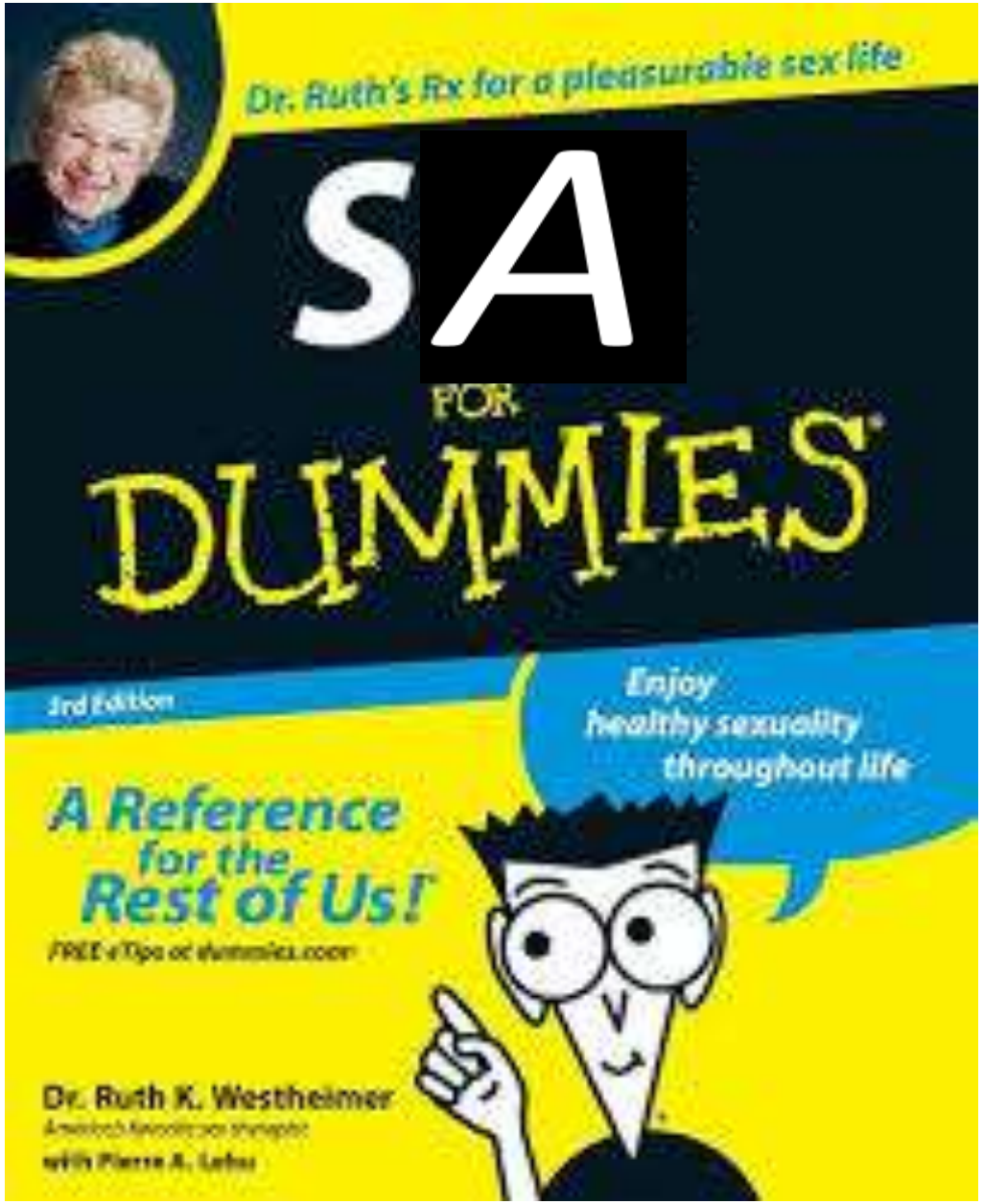
How would you test the scaffolding?

How coupled ladders are shaken in most of available literature



How to shake coupled ladders

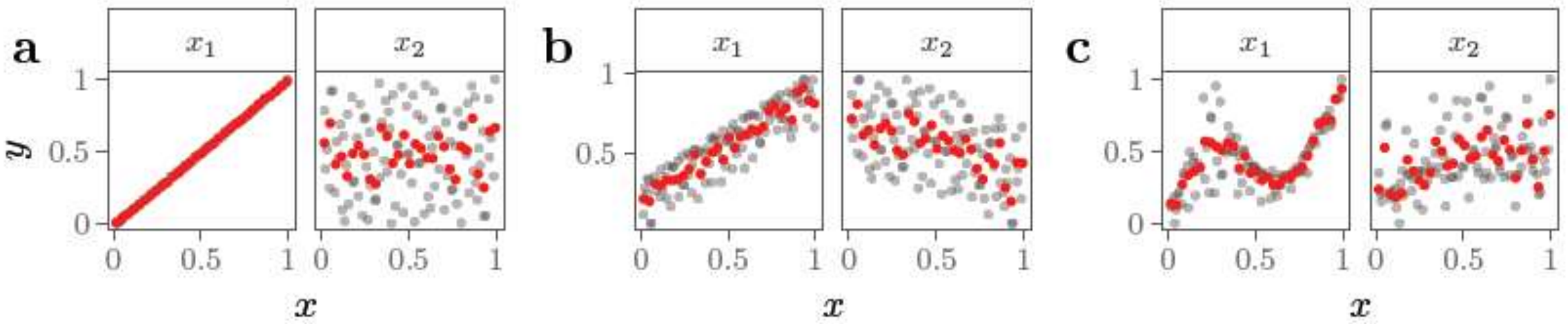




Sensitivity analysis made easy or "sensitivity analysis for dummies"

Open access paper

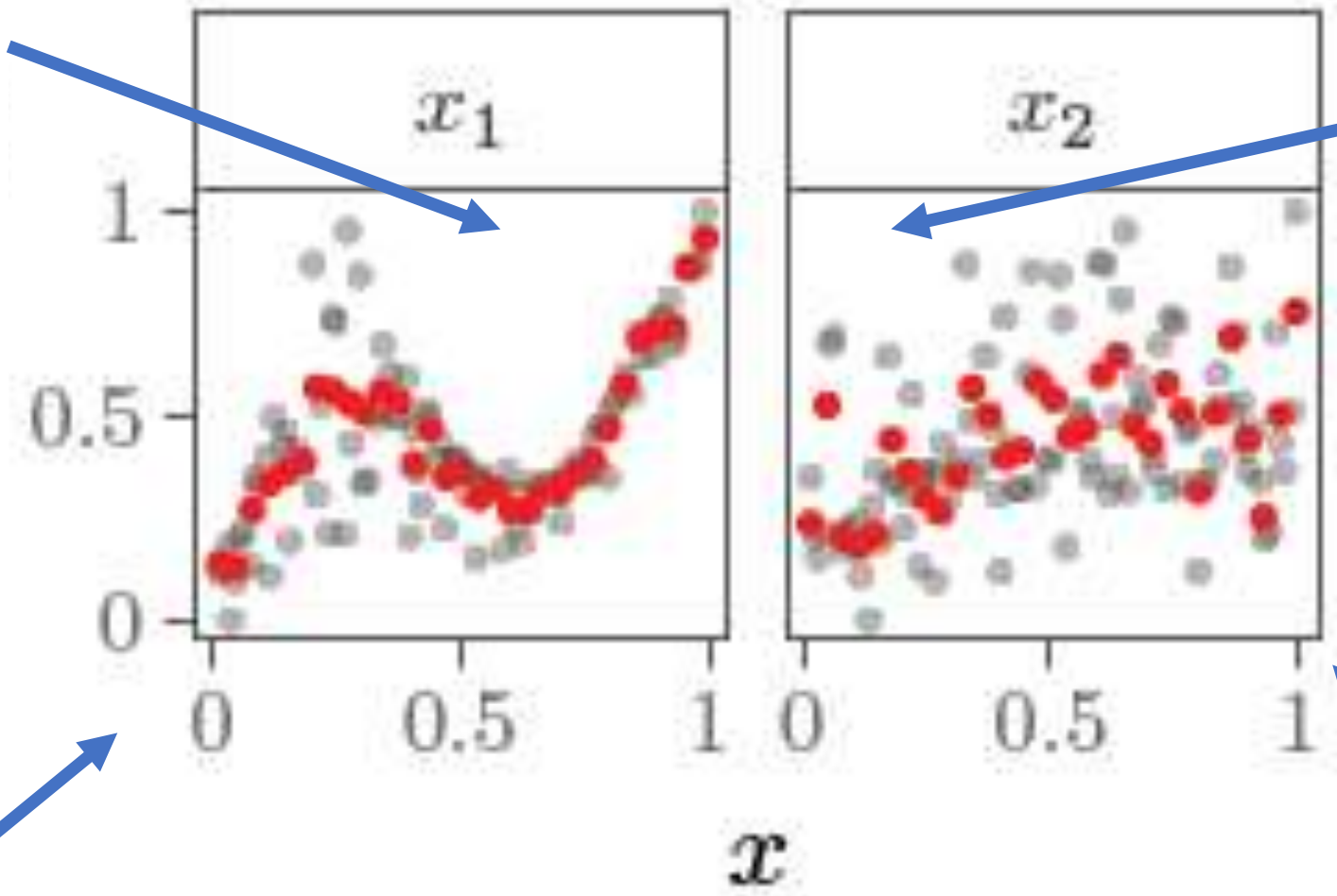
Do we need to compute indices? Can we do without statistics and calculus using the histograms we have met already?



'Stupid' histograms in the x_i, y plane, both in **[0,1]**, for different $y = f(x_i)$

Bigger ‘holes’

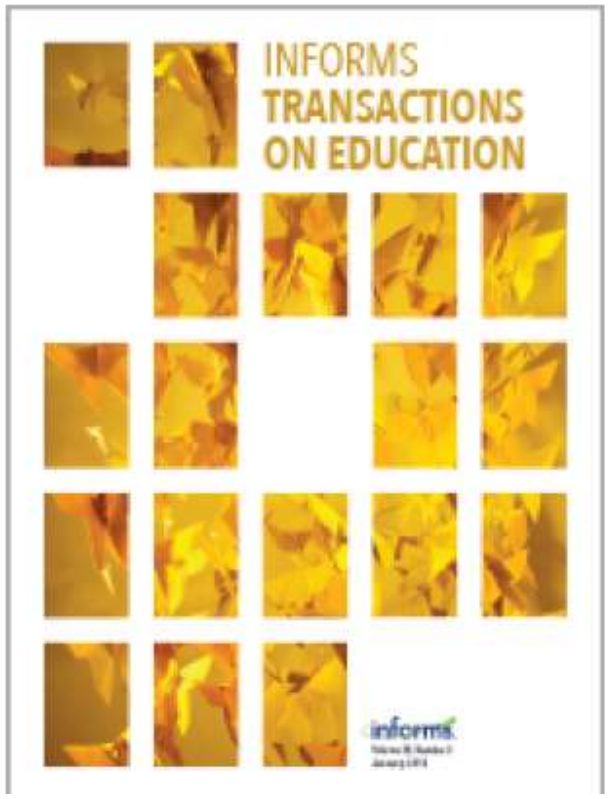
Smaller ‘holes’



=more important

=less important

Another way to bypass statistics and calculus



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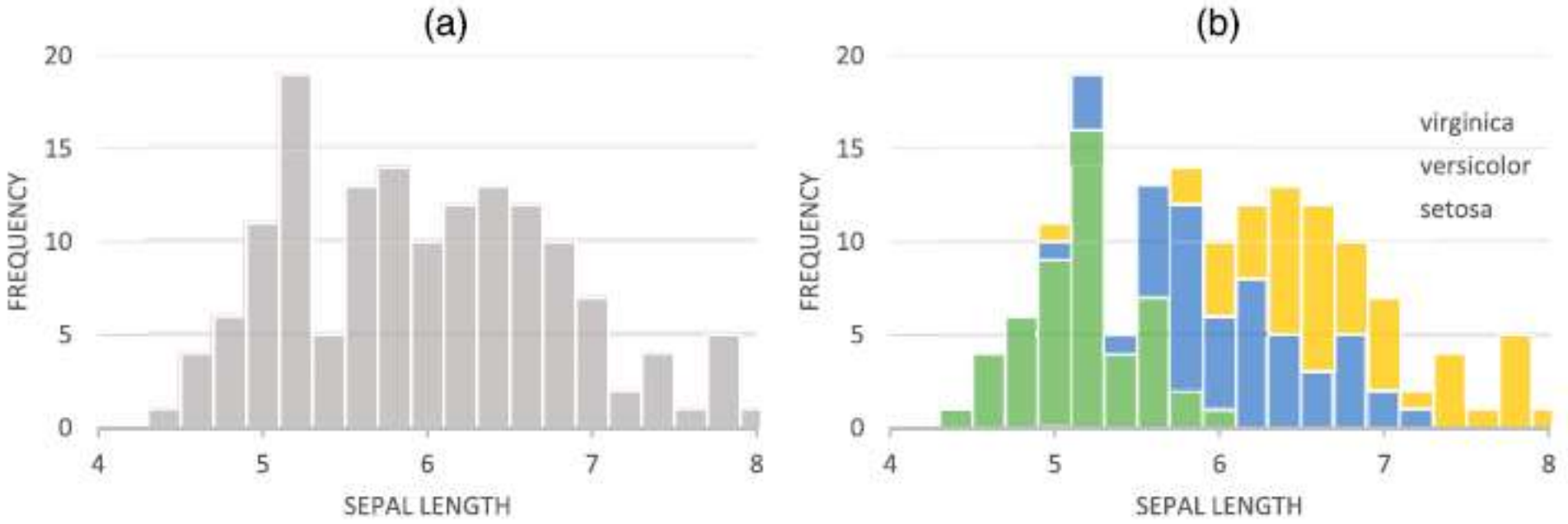
Monte Carlo Enhancement via Simulation Decomposition: A "Must-Have" Inclusion for Many Disciplines

Mariia Kozlova, Julian Scott Yeomans

Colouring the output histogram can give sensitivity insights …

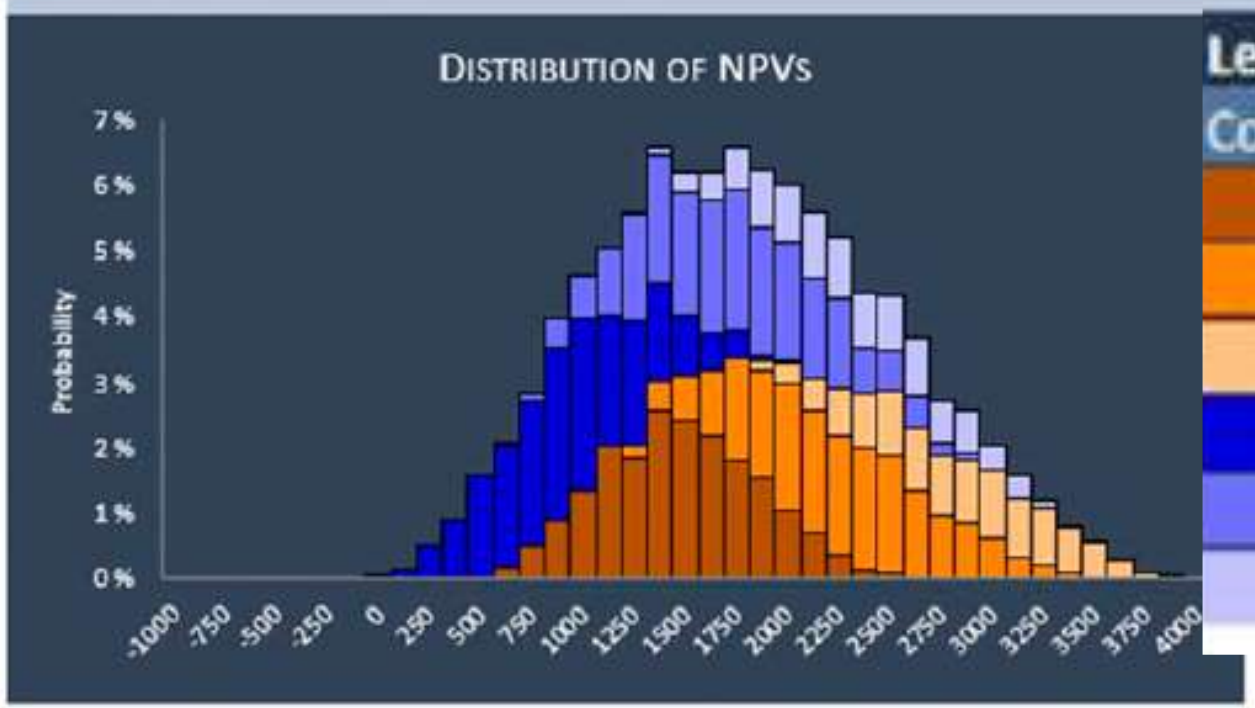


… without computing sensitivity indices

RESULTS

Min	100 €	Probability of negative NPV	0 %
Expected mean	**1 855 €**	Probability of positive NPV	100 %
Max	3 925 €	Standard deviation	713 €

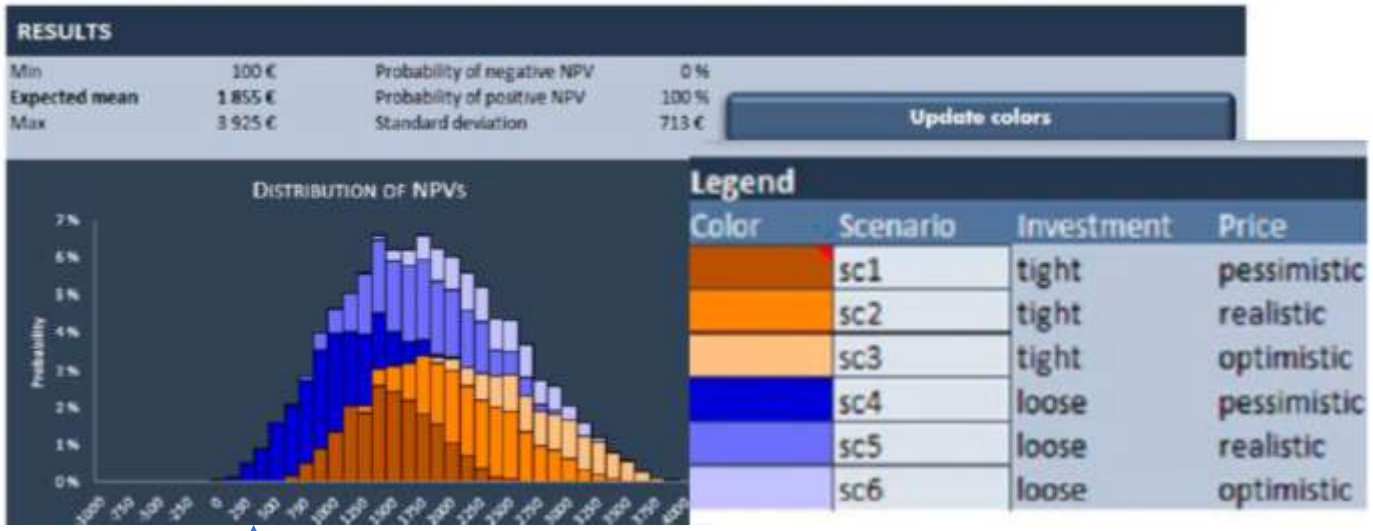
Update colors



Legend

Color	Scenario	Investment	Price
	sc1	tight	pessimistic
	sc2	tight	realistic
	sc3	tight	optimistic
	sc4	loose	pessimistic
	sc5	loose	realistic
	sc6	loose	optimistic

… without computing sensitivity indices

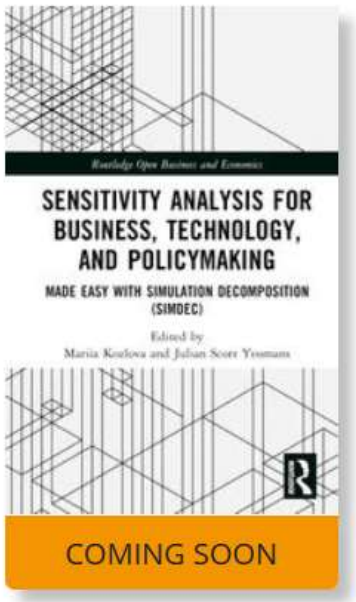


➔ The possibility of very low returns (dark blue) corresponds to loose investment and pessimistic prices

What is done here? We have two variables / options:

- Investment= ‘tight’ or ‘loose’
- Price=‘pessimistic’, ‘realistic’ or ‘optimistic’

Combing the 2 levels of investment with the three levels of price gives 2*3=6 ‘scenarios’



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1st Edition

Sensitivity Analysis for Business, Technology, and Policymaking

Made Easy with Simulation Decomposition (SimDec)

Edited By Mariia Kozlova, Julian Scott Yeomans

Hardback
£135.00

Don’t run the model just once

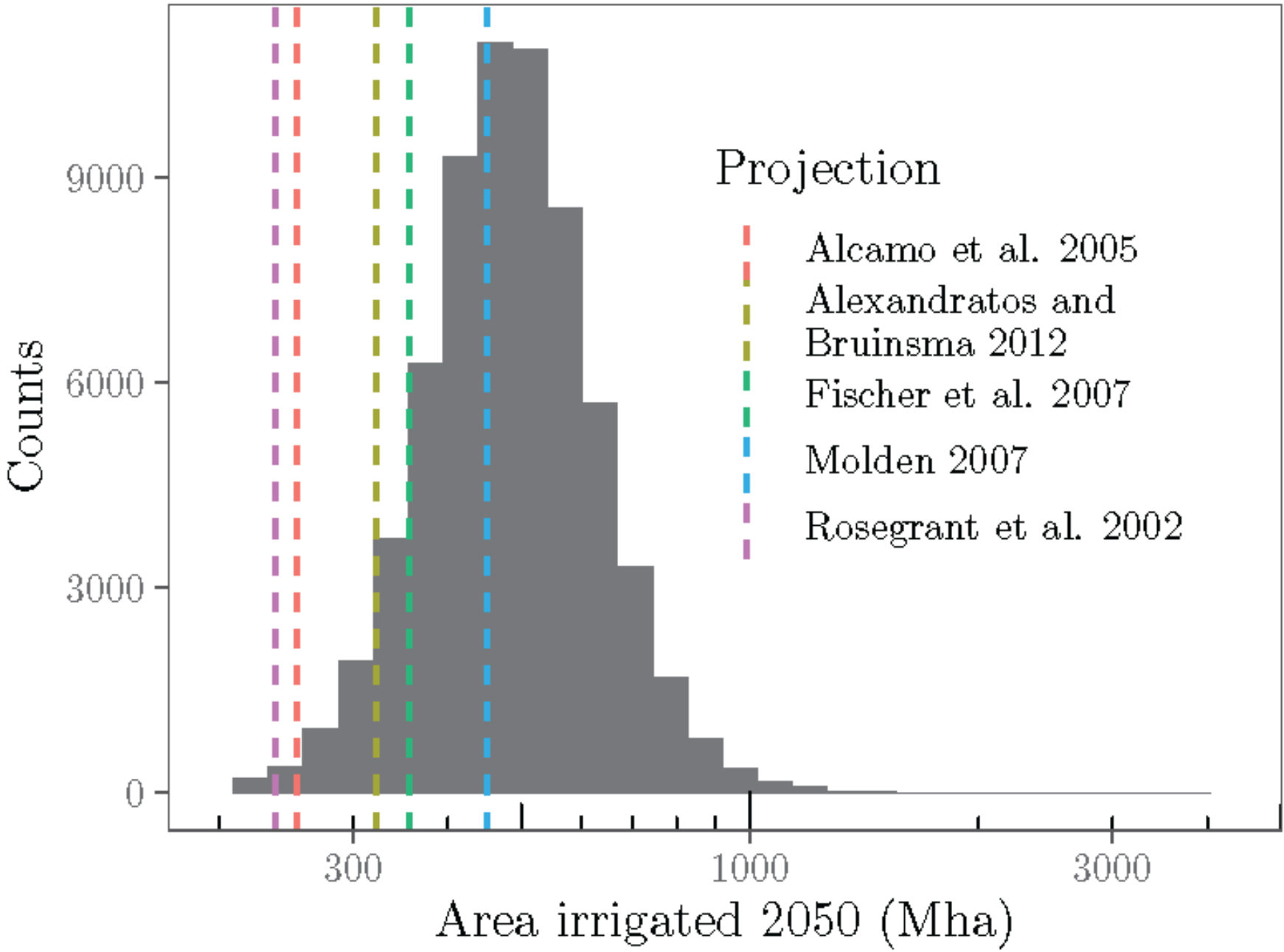
There is much to learn by running the model a few times, especially during model building

Lubarsky's Law of Cybernetic Entomology: there is always one more bug!

Model routinely used to produce point estimates may becomes non conservative when the uncertainty is plugged in

Current Models Underestimate Future Irrigated Areas

- How much land will need to be irrigated by the year 2050?
- Here the dashed lines represent deterministic model predictions from different models and datasets (from FAO & others organizations);
- An uncertainty analysis (grey histogram) reveals that the models are non-conservative: the need might be much larger



Citation:
Puy, A., Lo Piano, S., & Saltelli, A. (2020). Current models underestimate future irrigated areas. *Geophysical Research Letters*, *47*, e2020GL087360. https://doi.org/10.1029/2020GL087360

Don’t sample just parameters and boundary conditions

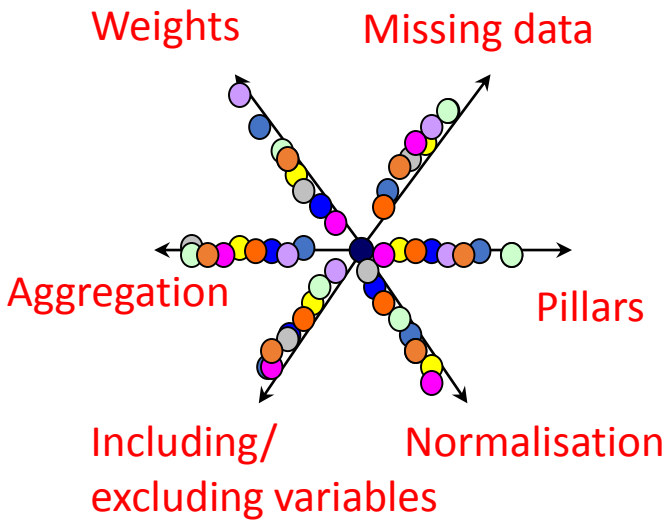
Explore thoroughly the space of the assumptions

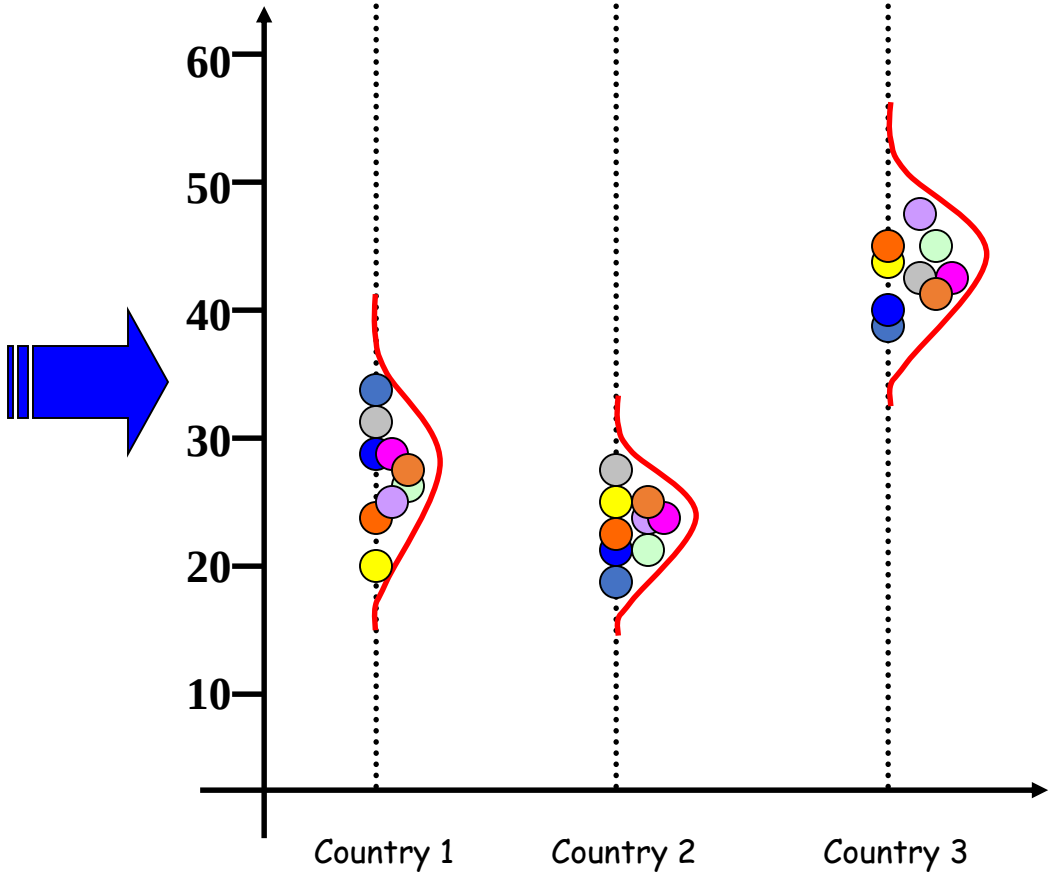
One can sample more than just factors:

- modelling assumptions,
- alternative data sets,
- resolution levels,
- scenarios …

Assumption	Alternatives
Number of indicators	▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	▪ original set of weights, ▪ factor analysis, ▪ equal weighting, ▪ data envelopment analysis
Aggregation rule	▪ additive, ▪ multiplicative, ▪ Borda multi-criterion

Space of alternatives



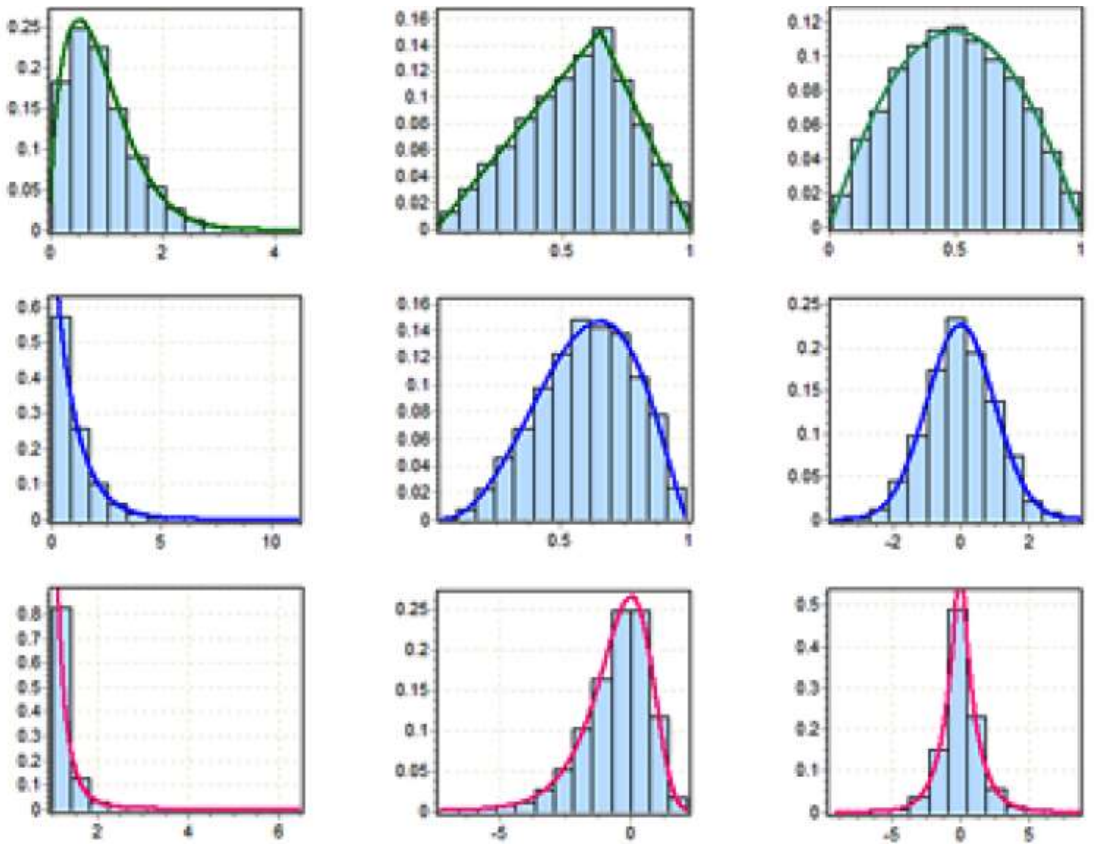


Building a Monte Carlo analysis

$$\begin{matrix} x_{11} & x_{12}\ldots & x_{1k} \\ x_{21} & x_{22}\ldots & x_{2k} \\ \ldots & \ldots & \ldots \\ x_{N1} & x_{N2} & x_{Nk1} \end{matrix}$$

Input matrix: each column is a sample of size N from the distribution of a factor

Each row is a sample trial of size k to generate a value of y



Examples of distributions of input factors

NEVER vary all factors of the same amount

Be it 5%, 10%, or 20%

New WHO estimates: Up to 190 000 people could die of COVID-19 in Africa if not controlled

07 May 2020

Brazzaville – Eighty-three thousand to 190 000 people in Africa could die of COVID-19 and 29 million to 44 million could get infected in the first year of the pandemic if containment measures fail, a new study by the World Health Organization (WHO) Regional Office for Africa finds. The research, which is based on prediction modelling, looks at 47 countries in the

Speculative scenario in which ten uncertain input probabilities are increased by an arbitrary 10% — as if they were truly equally uncertain — with no theoretical or empirical basis for such a choice

World Health Organization
REGIONAL OFFICE FOR Africa

In a numerical experiment relating to a real-life application the range of uncertainty of each input is crucial input to the analysis, and often the most expensive to get

Conclusions

"But the real strength of the models, in my mind at least, were in sensitivity analysis (where one could examine the response of the model to parameters or structures that were not known with precision (i.e., sensitivity analysis), and in the examination of the behavior of the model components relative to that of the real system in question (i.e., validation)."

Hall, C. A. S. (2020). Systems Ecology and Limits to Growth: History, Models, and Present Status. In G. S. Metcalf, K. Kijima, & H. Deguchi, eds., *Handbook of Systems Sciences*, Singapore: Springer, , pp. 1–38.

… By undertaking sensitivity analysis and validation, a great deal can be learned about the real system, including what you do not know. (Hall, 2020)

Hall, C. A. S. (2020). Systems Ecology and Limits to Growth: History, Models, and Present Status. In G. S. Metcalf, K. Kijima, & H. Deguchi, eds., *Handbook of Systems Sciences*, Singapore: Springer, , pp. 1–38.

The End

https://mstdn.social/@AndreaSaltelli/