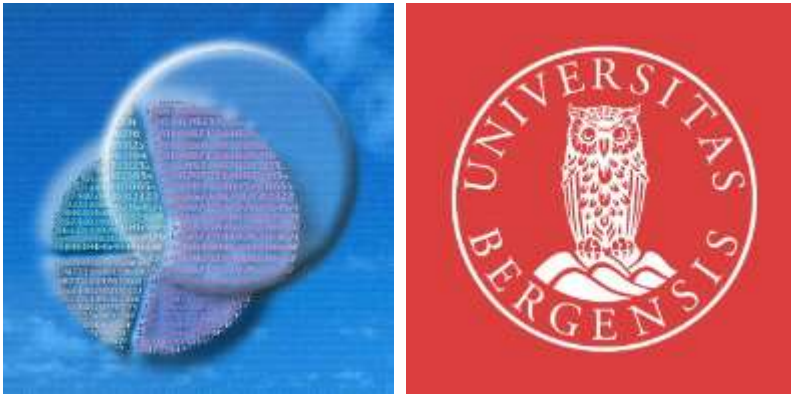




# Why sensitivity analysis (or: How not to do a sensitivity analysis)

Andrea Saltelli



Course MNF990, Theory of Science and Ethics at the  
Faculty of Mathematics and Natural Sciences  
University of Bergen, September 27, 2024

# Where to find this talk: [www.andreasaltelli.eu](http://www.andreasaltelli.eu)

Andrea Saltelli

HOME ABOUT ME PUBLICATIONS NEWS & VIDEOS RESOURCES

## August 25 2023: The politics of modelling is out!

**the politics of modelling**  
numbers between science and policy  
edited by Andrea Saltelli & Monica Di Fiore  
OXFORD

### Praise for the volume

*"A long awaited examination of the role—and obligation—of modeling."*  
**Nassim Nicholas Taleb**, Distinguished Professor of Risk Engineering, NYU Tandon School of Engineering. Author, of the 5 -volume series *Incerto*.

\*\*\*

*"A breath of fresh air and a much needed cautionary view of the ever-widening dependence on mathematical modeling."*  
**Orrin H. Pilkey**, Professor at Duke University's Nicholas School of the Environment, co-author with Linda Pilkey-Jarvis of *Useless Arithmetic: Why Environmental Scientists Can't Predict the Future*, Columbia University Press 2009.

\*\*\*

*"The methods by which power insinuates itself into models, and facilitates their portability and*

### Mastodon Toots by @AndreaSaltelli

AndreaSaltelli  
2024/2/16 16:38

Are you interested in sensitivity analysis?  
Here a short story of it.  
Open access, online in draft form:  
[sciedirect.com/science/arti](https://sciedirect.com/science/arti)

Andrea

Environmental Modelling & Software  
An annotated timeline of sensitivity analysis

View on [mstdn.social](https://mstdn.social)

But the real strength of the models, in my mind at least, were in sensitivity analysis (where one could examine the response of the model to parameters or structures that were not known with precision (i.e., sensitivity analysis), and in the examination of the behavior of the model components relative to that of the real system in question (i.e., validation).

Hall, C. A. S. (2020). Systems Ecology and Limits to Growth: History, Models, and Present Status. In G. S. Metcalf, K. Kijima, & H. Deguchi, eds., *Handbook of Systems Sciences*, Singapore: Springer, , pp. 1–38.



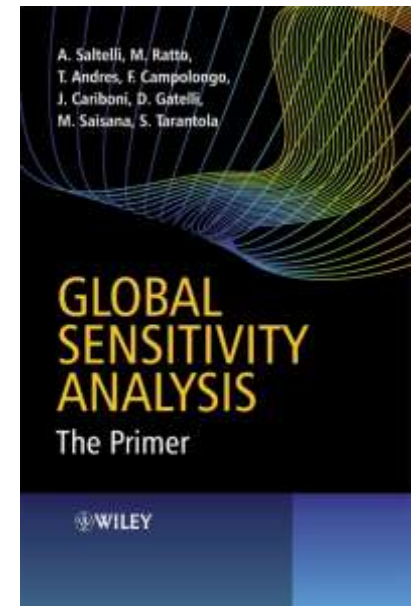
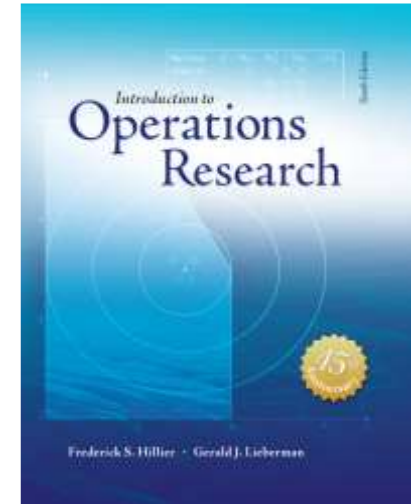
… By undertaking sensitivity analysis and validation, a great deal can be learned about the real system, including what you do not know. (Hall, 2020)

Hall, C. A. S. (2020). Systems Ecology and Limits to Growth: History, Models, and Present Status. In G. S. Metcalf, K. Kijima, & H. Deguchi, eds., *Handbook of Systems Sciences*, Singapore: Springer, , pp. 1–38.

# Linear programming and sensitivity analysis

Linear programming viewpoint: testing which parameter, when changed in isolation, lead to a change in the optimal solution

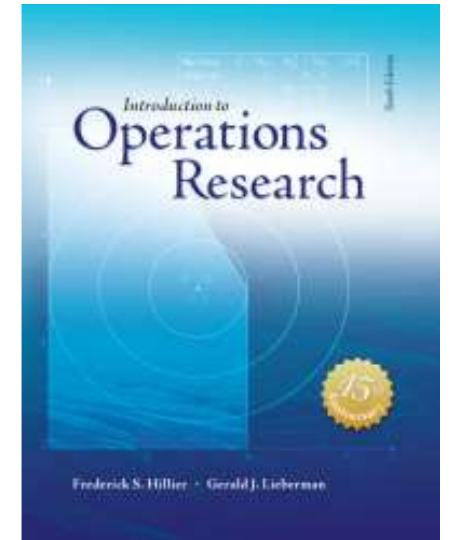
Global SA viewpoints: explore the distribution of the optimal solution when all uncertain coefficients are allowed to vary over their plausible range



We should not be surprised that the sensitivity analysis practiced in linear programming is linear!

Yet so much can be lost by neglecting that part of the uncertainty that escapes linearity

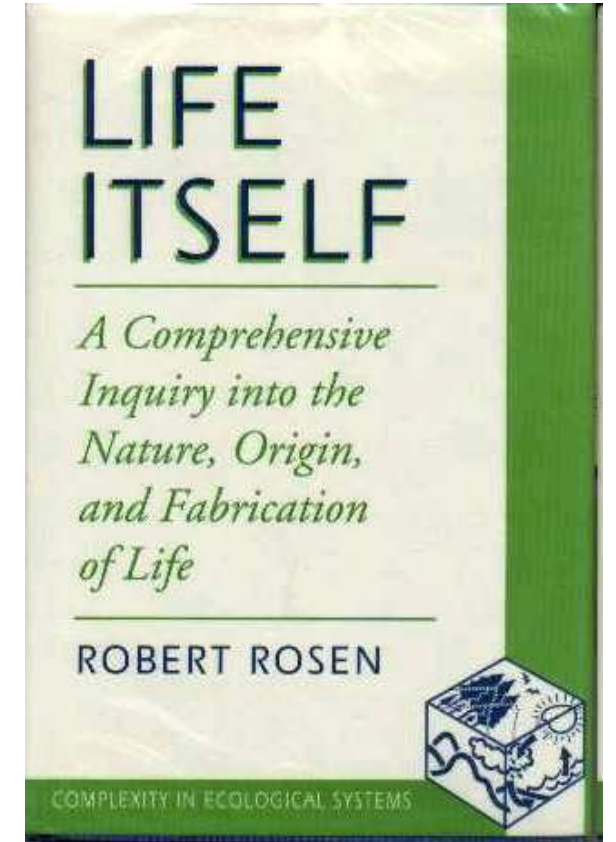
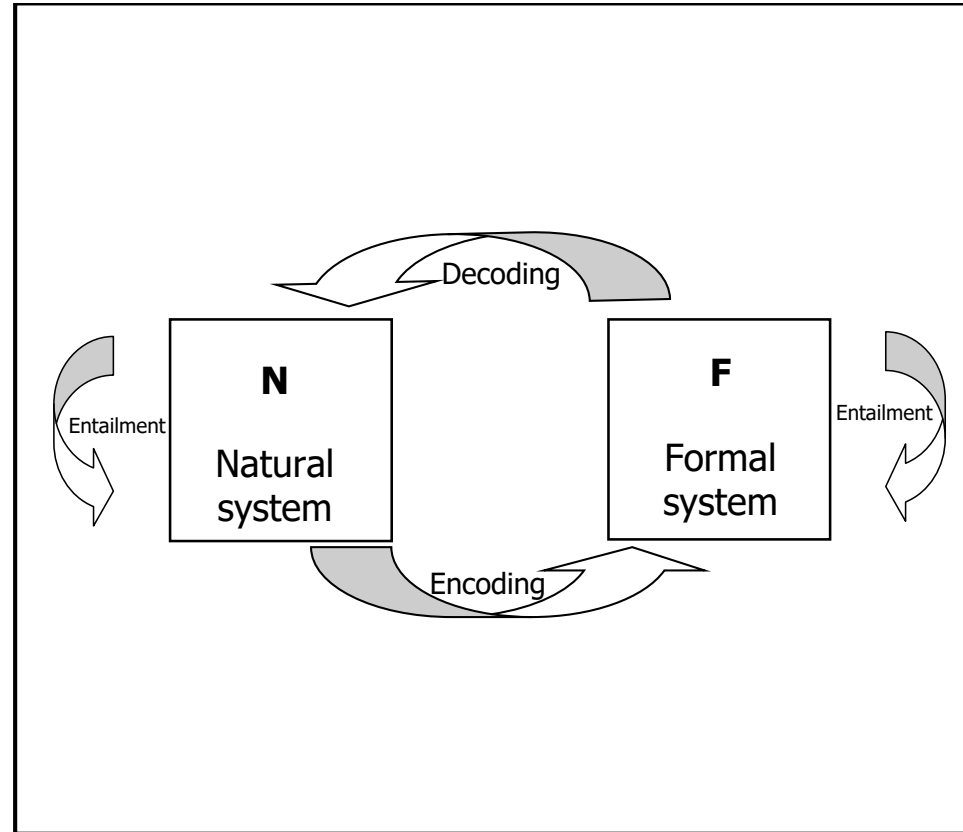
The advantages of understating global methods for uncertainty and sensitivity analysis are very large, including the possibility to test to flexibility of managerial decision when ‘all the rest’ is varying as well



Modelling is a craft more than a science



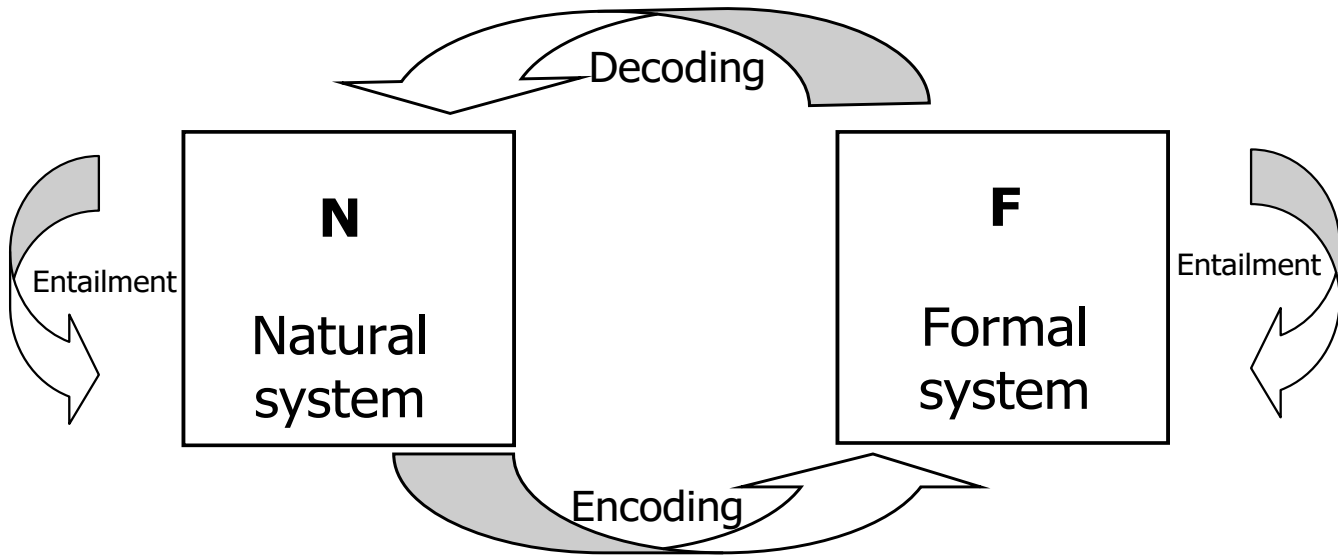
# Modelling as a craft rather than as a science for Robert Rosen



R. Rosen, *Life Itself: A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life*. Columbia University Press, 1991.

Louie, A.H. 2010. "Robert Rosen's Anticipatory Systems." Edited by Riel Miller. *Foresight* 12 (3): 18–29. <https://doi.org/10.1108/14636681011049848>.





What is a model ?



Robert Rosen  
(1934-1998)

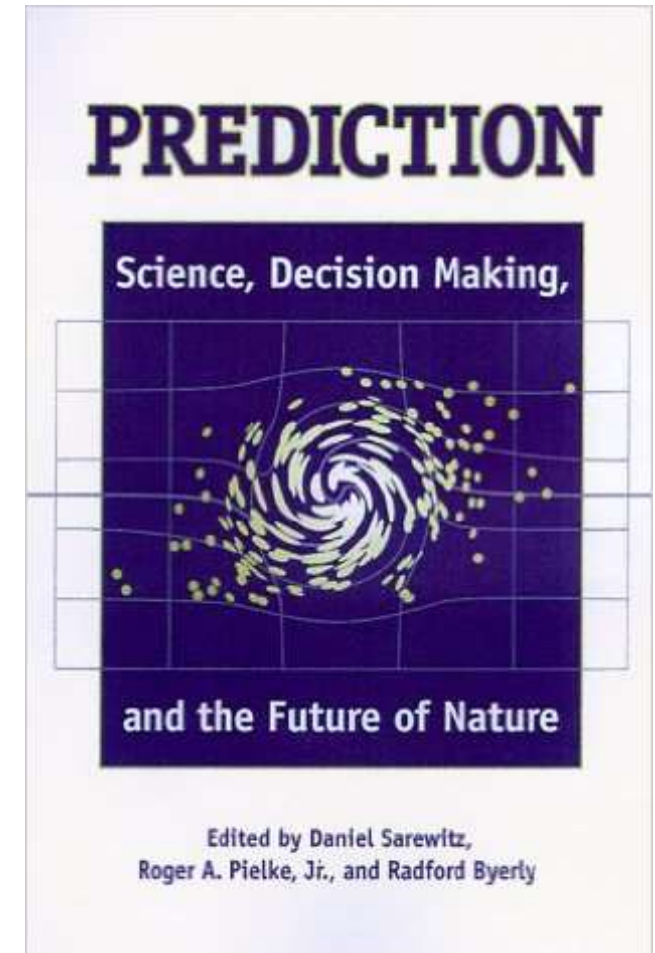
“models are most useful when they are used to challenge existing formulations, rather than to validate or verify them”



Naomi Oreskes

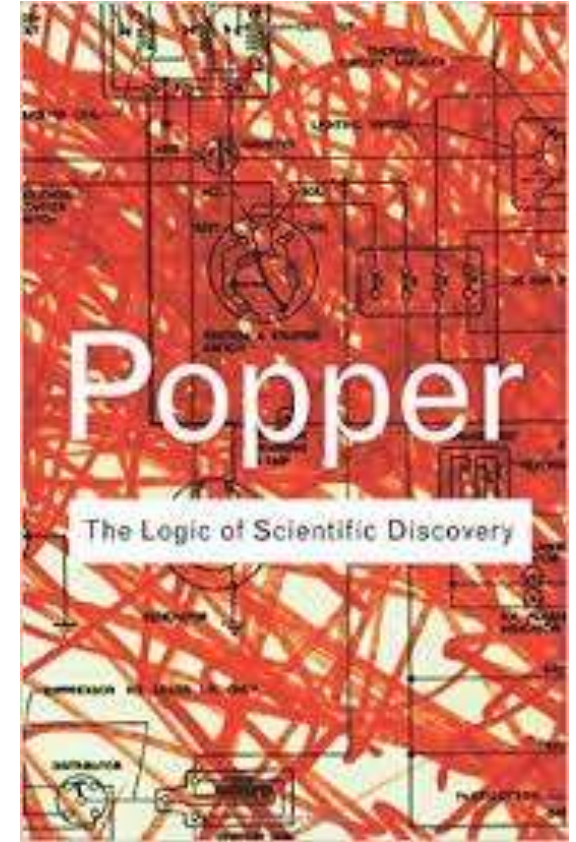
N. Oreskes, K. Shrader-Frechette, and K. Belitz, “Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences,” *Science*, 263, no. 5147, 1994.

# Models are not physical laws

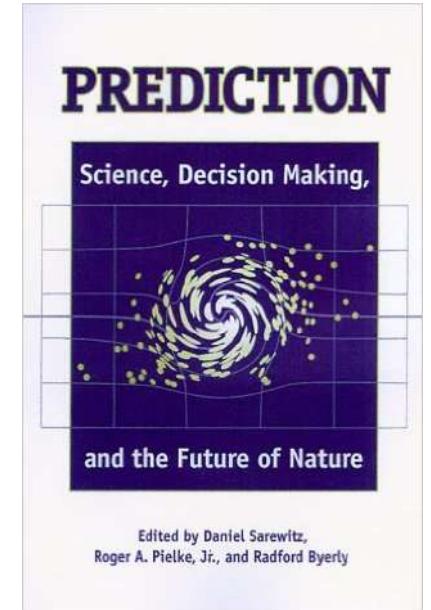


Oreskes, N., 2000, Why predict? Historical perspectives on prediction in Earth Science, in Prediction, Science, Decision Making and the future of Nature, Sarewitz et al., Eds., Island Press, Washington DC

“[...] to be of value in theory testing, the predictions involved must be capable of refuting the theory that generated them”  
(N. Oreskes)



“When a model generates a prediction, of what precisely is the prediction a test? The laws? The input data? The conceptualization?”

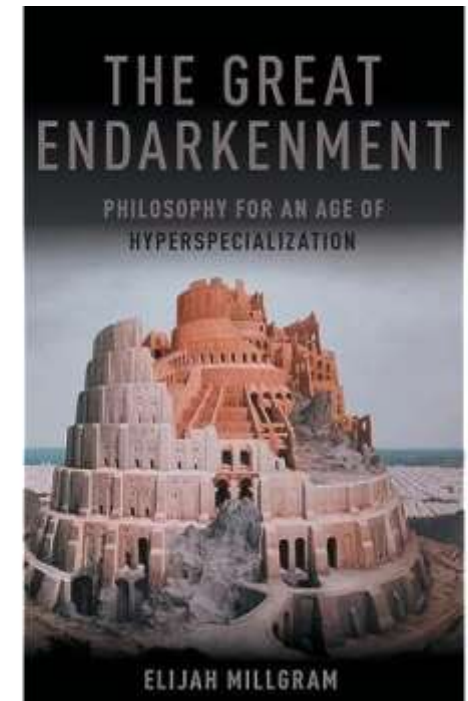


Any part (or several parts) of the model might be in error, and there is no simple way to determine which one it is”

Models have little memory

“[...] The process of constructing and validating [value-at risk] models is time consuming and detail oriented; normally even the people who produced the model will not remember many of the assumptions incorporated into it, short of redoing their work, which means that the client cannot simply ask them what went into it.”

E. Millgram *The Great Endarkenment*, p. 29





# Caeteris are never paribus

Ceteris paribus or caeteris paribus (Latin) = "all other things being equal" or "other things held constant" or "all else unchanged"

# The case of DSGE, dynamic stochastic general equilibrium models

Rational expectations of agents  
Efficient market hypothesis

Philip Mirowski



Philip Mirowski, 2013, Never let a serious crisis go wasted, Verso Books.

The US senate and Queen Elisabeth perplexed...



Philip Mirowski, 2013, Never let a serious crisis go wasted, Verso Books.

# Dangers of mathematization of economics



Wolfgang Drechsler



Erik S. Reinert



Paul Romer



Philip Mirowski

W. Drechsler, “On the possibility of quantitative–mathematical social science, chiefly economics,” *J. Econ. Stud.*, vol. 27, no. 4/5, pp. 246–259, 2000.

E. S. Reinert, “Full circle: economics from scholasticism through innovation and back into mathematical scholasticism,” *J. Econ. Stud.*, vol. 27, no. 4/5, pp. 364–376, Aug. 2000.

P. Romer, “Mathiness in the Theory of Economic Growth,” *Am. Econ. Rev.*, vol. 105, no. 5, pp. 89–93, May 2015.

Mirowski, Philip. 2013. *Never Let a Serious Crisis Go to Waste: How Neoliberalism Survived the Financial Meltdown*. Verso.

Don't confuse the map with the territory

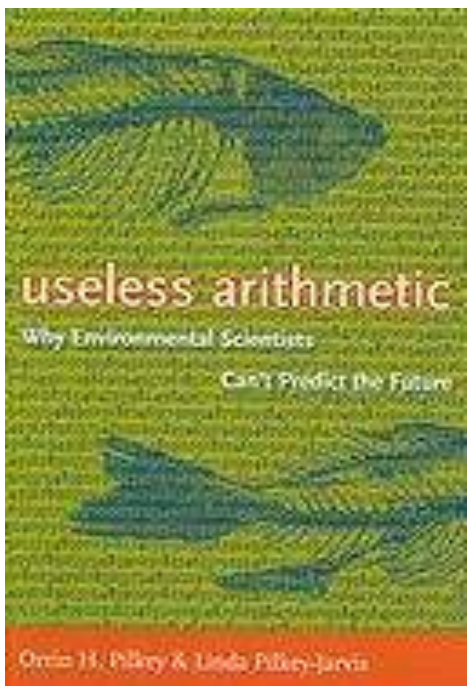
If you do, sensitivity analysis will not save you



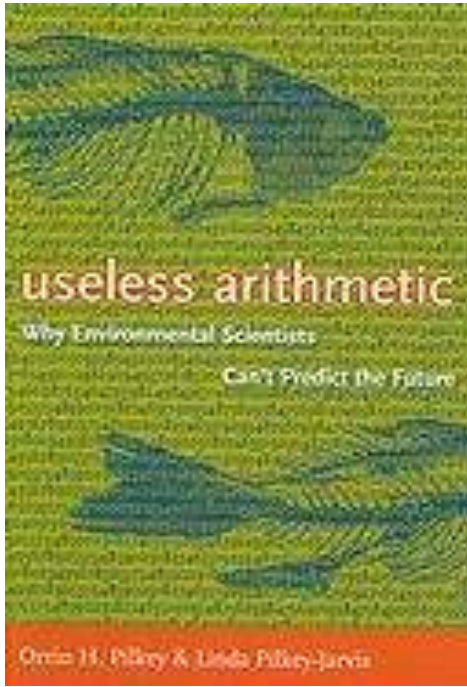


Orrin H. Pilkey

<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature>>



Useless Arithmetic: Why Environmental Scientists Can't Predict the Future  
by Orrin H. Pilkey and Linda Pilkey-Jarvis, Columbia University Press, 2009.

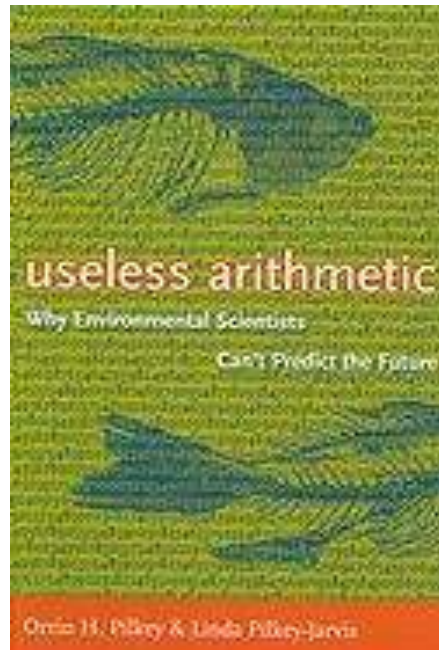


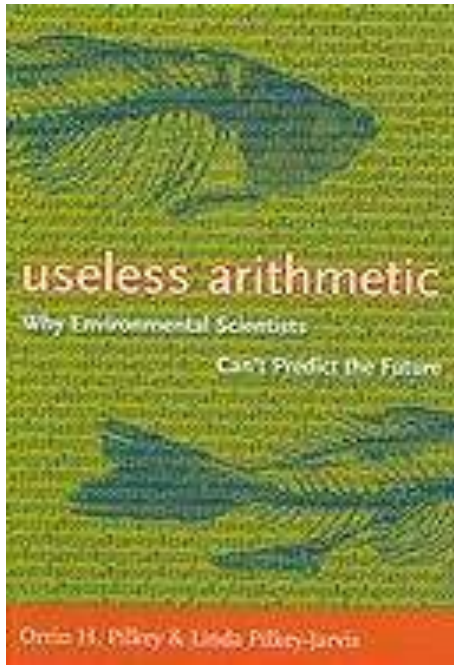
<<...If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit>>



One of the examples discussed concerns the **Yucca Mountain** repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

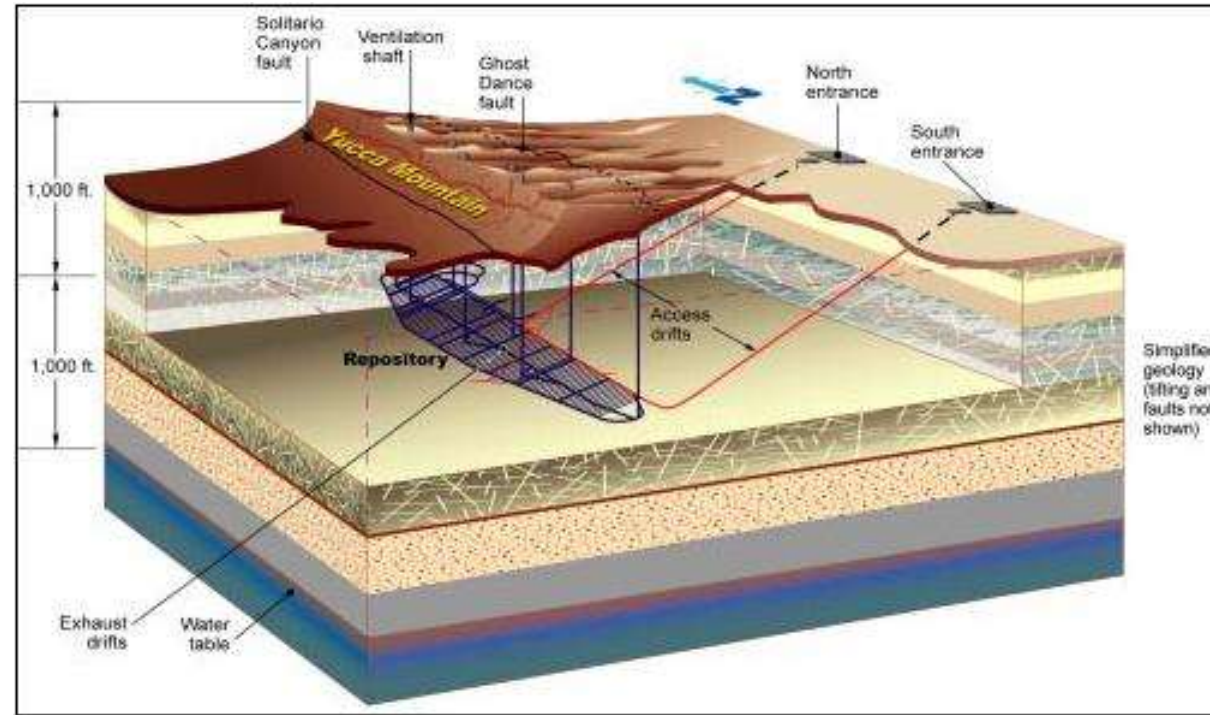
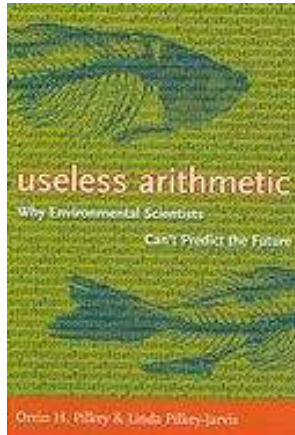
TSPA is Composed of 286 sub-models.





TSPA (like any other model) **relies on assumptions** → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal





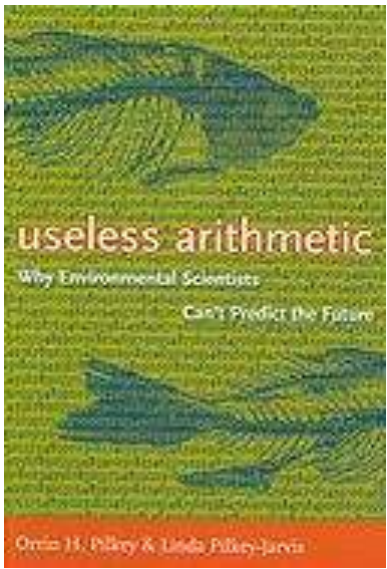
The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter (the  $^{36}\text{Cl}$  story)



In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→... SA useless if it is instead ~ 3,000 millimetres per year.





“Scientific mathematical modelling should involve constant efforts to falsify the model”

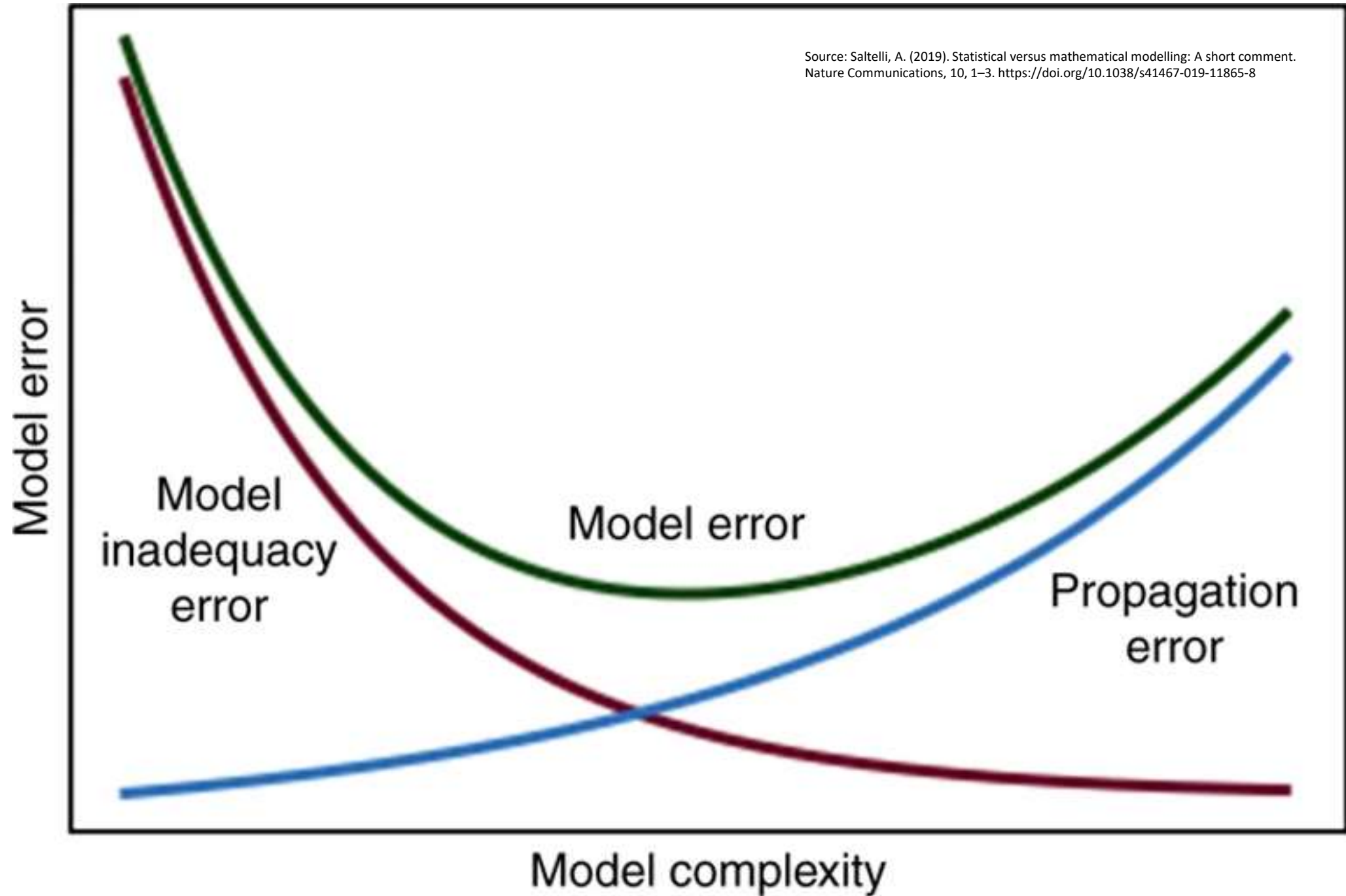
→ Organized skepticism (as per CUDOS)

Communalism, Universalism, Disinterestedness, Organized Skepticism, from sociology of science, Robert K. Merton.

Beware the size of your model

Mind the conjecture of O'Neil

Source: Saltelli, A. (2019). Statistical versus mathematical modelling: A short comment. Nature Communications, 10, 1–3. <https://doi.org/10.1038/s41467-019-11865-8>





Conjecture by O’Neill, also known as Zadeh’s principle of incompatibility, whereby as complexity increases “precision and significance (or relevance) become almost mutually exclusive characteristics”

In M. G. Turner and R. H. Gardner, “Introduction to Models” in *Landscape Ecology in Theory and Practice*, New York, NY: Springer New York, 2015, pp. 63–95.

L. Zadeh, “Outline of a New Approach to the Analysis of Complex Systems and Decision Processes,” *IEEE Trans. Syst. Man. Cybern.*, vol. 3, no. 1, pp. 28–44, 1973.

Puy, Arnald, Pierfrancesco Beneventano, Simon A. Levin, Samuele Lo Piano, Tommaso Portaluri, and Andrea Saltelli. 2022. “Models with Higher Effective Dimensions Tend to Produce More Uncertain Estimates.” *Science Advances* 8 (eabn9450).

Simple principles of responsible modelling

# Mind the assumptions

Assess uncertainty and sensitivity

# Mind the hubris

Complexity can be the enemy of relevance

# Mind the framing

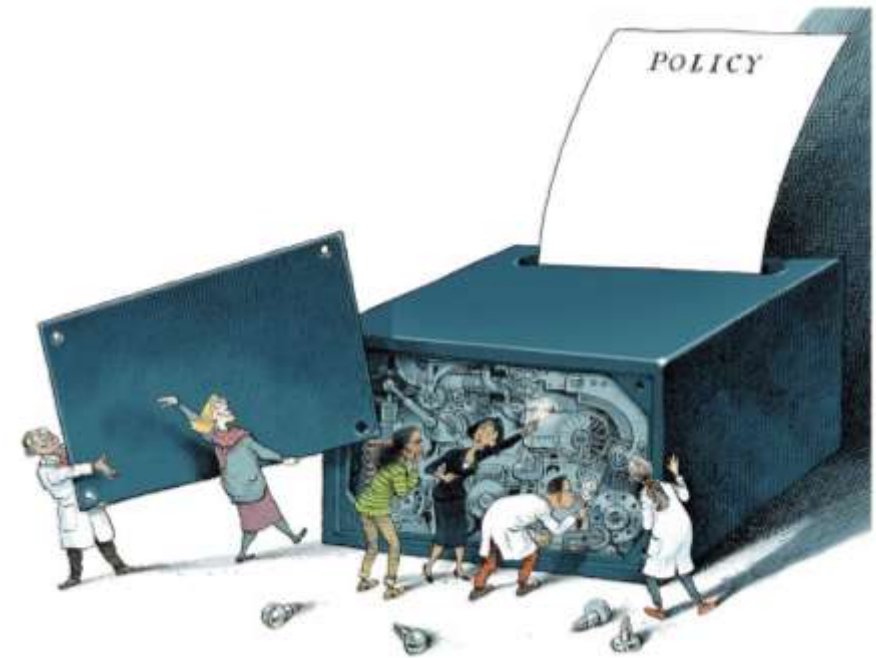
Match purpose and context

# Mind the consequences

Quantification can backfire.

# Mind the unknowns

Acknowledge ignorance



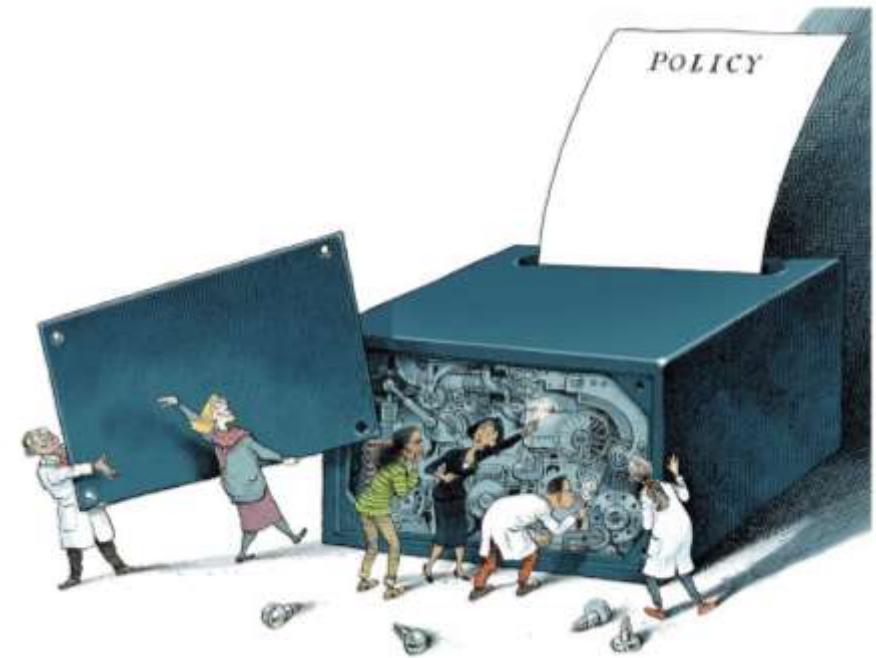
Source: A. Saltelli, G. Bammer, I. Bruno, et al., Five ways to ensure that models serve society: a manifesto, Nature 582 (2020) 482–484.

# Mind the assumptions

Assess uncertainty and sensitivity

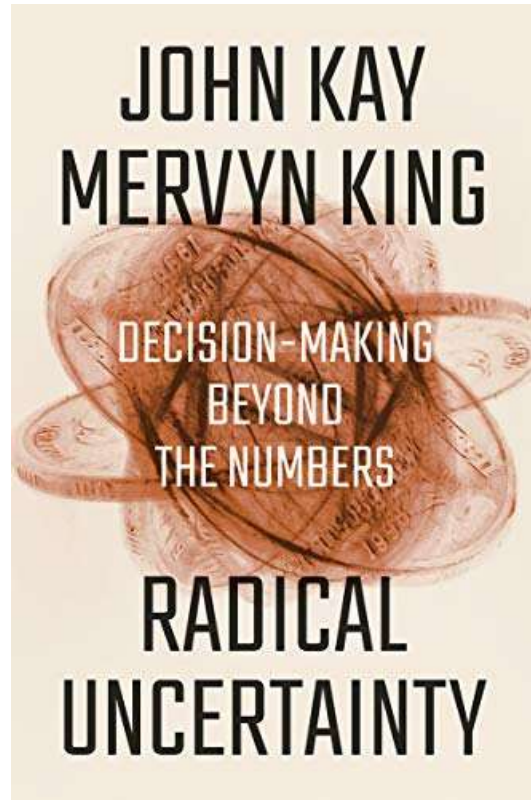


... models require input values for which there is no reliable information...



Source: A. Saltelli, G. Bammer, I. Bruno, et al., Five ways to ensure that models serve society: a manifesto, *Nature* 582 (2020) 482–484.

Models ask as input information which we don't have –  
The case of WEBTAG



John Kay

## WebTAG: Annual Percentage Change in Car Occupancy (% pa) up to 2036

Journey Purpose	Weekday					Weekend	All Week
	7am-10am	10am-4pm	4pm-7pm	7pm-7am	Weekday Average		
Work	-0.48	-0.4	-0.62	-0.5	-0.44	-0.48	-0.45
Non - Work (commuting and other)	-0.67	-0.65	-0.53	-0.47	-0.59	-0.52	-0.56

Source: J. A. Kay, “Knowing when we don’t know,” 2012,  
[https://www.ifs.org.uk/docs/john\\_kay\\_feb2012.pdf](https://www.ifs.org.uk/docs/john_kay_feb2012.pdf)



# Mind the assumptions

Assess uncertainty and sensitivity

# Mind the hubris

Complexity can be the enemy of relevance

# Mind the framing

Match purpose and context

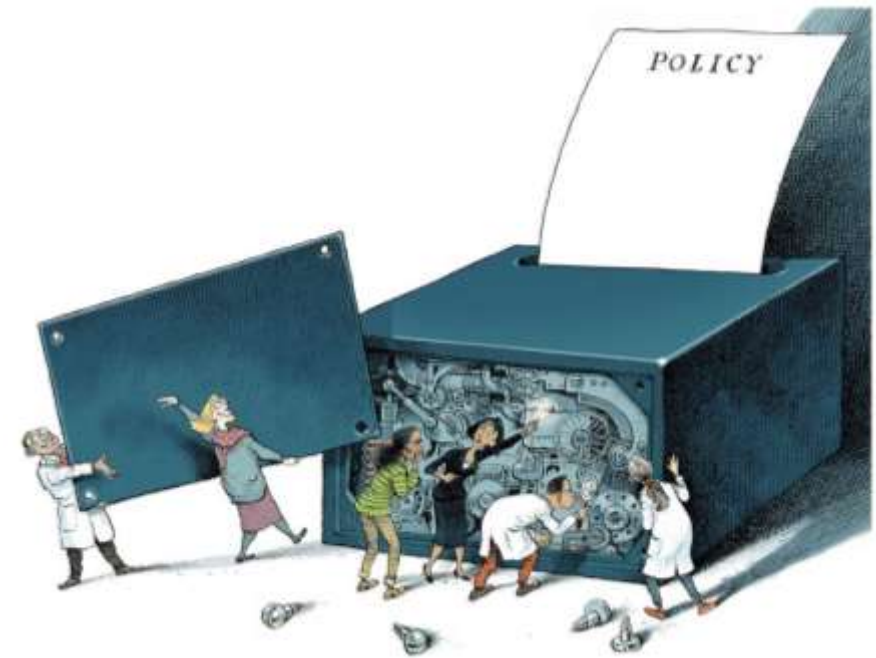


# Mind the consequences

Quantification can backfire.

# Mind the unknowns

Acknowledge ignorance



Source: A. Saltelli, G. Bammer, I. Bruno, et al., Five ways to ensure that models serve society: a manifesto, Nature 582 (2020) 482–484.



# Mind the consequences

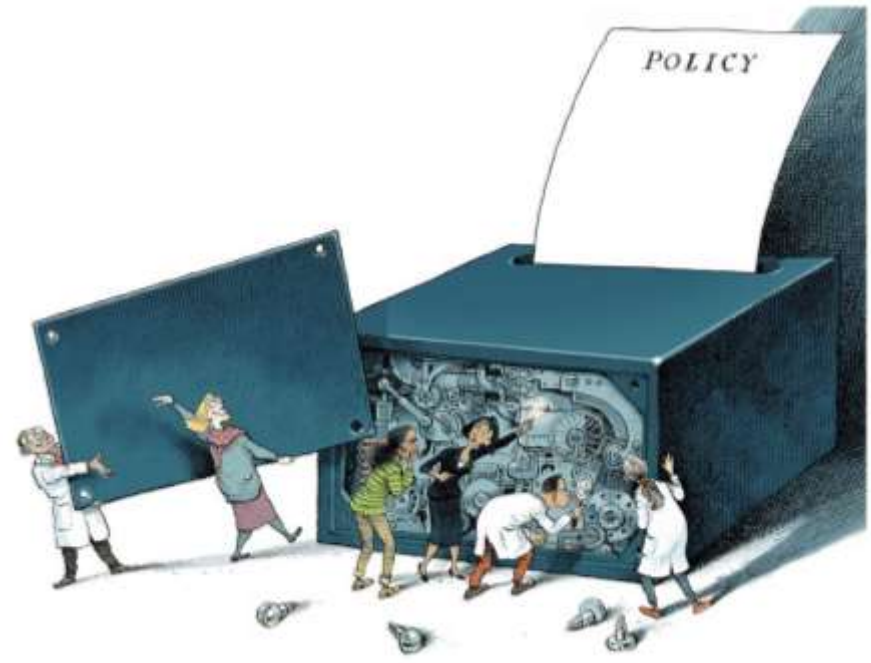
Quantification can backfire.

[← Back to Article](#)

WIRED MAGAZINE: 17.03

## Recipe for Disaster: The Formula That Killed Wall Street

By Felix Salmon 02.23.09



$$\Pr[T_A < 1, T_B < 1] = \Phi_2(\Phi^{-1}(F_A(1)), \Phi^{-1}(F_B(1)), \gamma)$$

Here's what killed your 401(k) *David X. Li's Gaussian copula function as first published in 2000. Investors exploited it as a quick—and fatally flawed—way to assess risk. A shorter version appears on this month's cover of Wired.*

Here is what killed your 401(k)...

Li's Gaussian copula function ...

**Nassim Nicholas Taleb**, hedge fund manager and author of *The Black Swan*, is particularly harsh when it comes to the copula. "People got very excited about the Gaussian copula because of its mathematical elegance, but the thing never worked," he says. "Co-association between securities is not measurable using correlation," because past history can never prepare you for that one day when everything goes south. "Anything that relies on correlation is charlatanism."

Felix Salmon, Wired, February 2009

**W I R E D**

Source: <https://www.wired.com/2009/02/wp-quant/>

# Mind the assumptions

Assess uncertainty and sensitivity

# Mind the hubris

Complexity can be the enemy of relevance

# Mind the framing

Match purpose and context

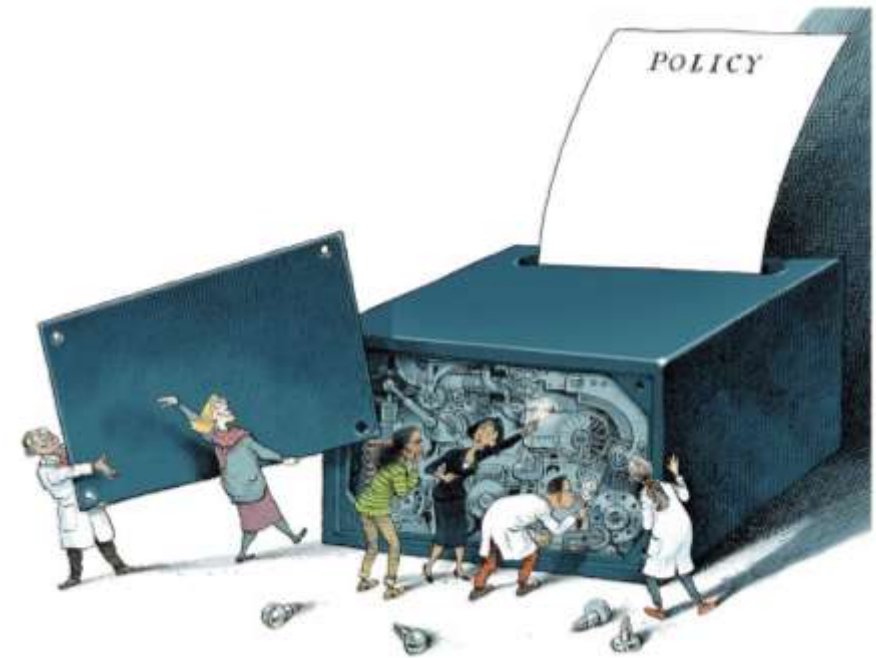
# Mind the consequences

Quantification can backfire.



# Mind the unknowns

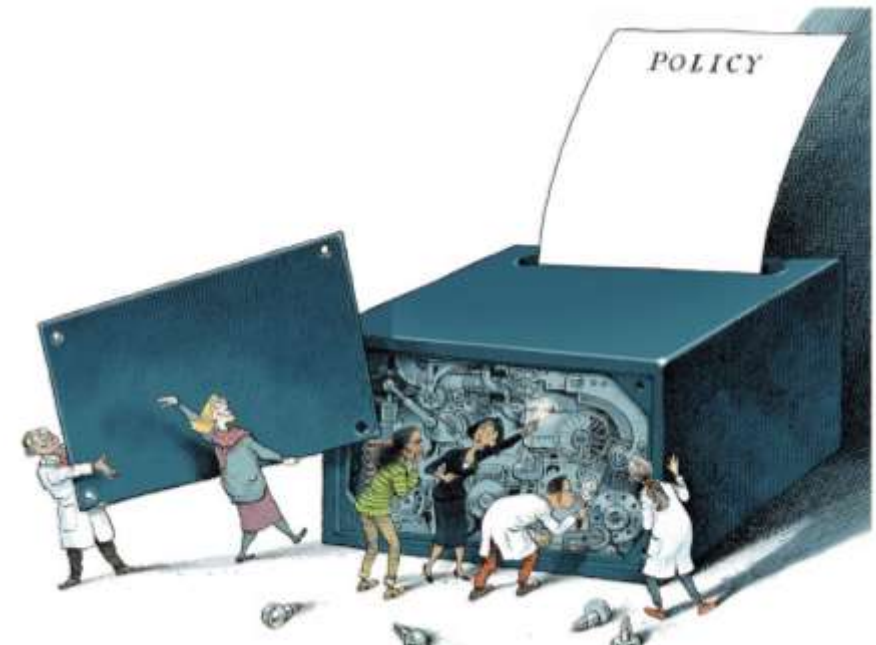
Acknowledge ignorance



Source: A. Saltelli, G. Bammer, I. Bruno, et al., Five ways to ensure that models serve society: a manifesto, Nature 582 (2020) 482–484.

# Mind the unknowns

## Acknowledge ignorance



Source: A. Saltelli, G. Bammer, I. Bruno, et al., Five ways to ensure that models serve society: a manifesto, Nature 582 (2020) 482–484.

From Socrates's "knowing of not knowing" to Nicolaus Cusanus' Docta Ignorantia, ignorance was a virtue until Descartes

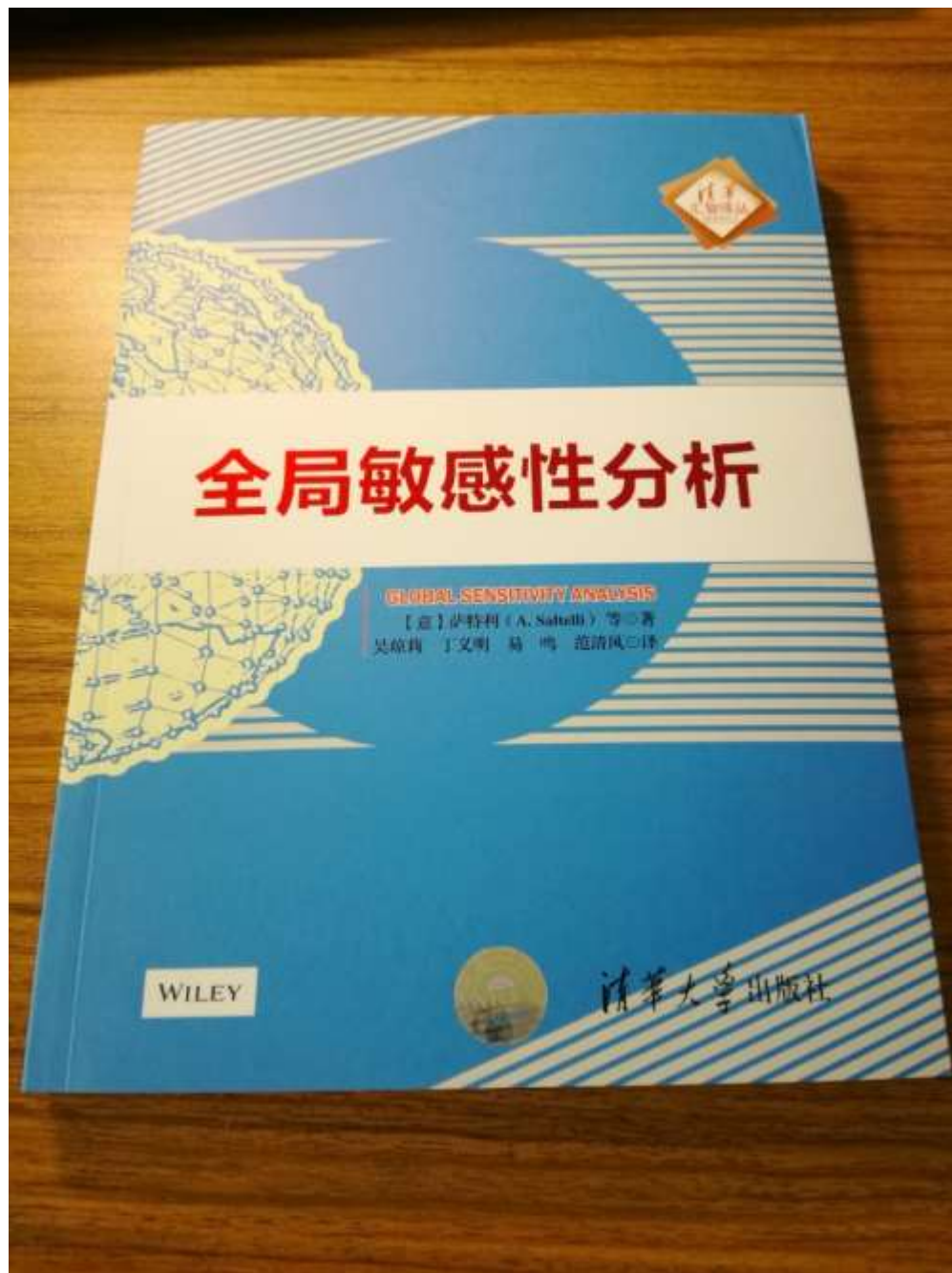


A. Saltelli, M. Ratto,  
T. Andres, F. Campolongo,  
J. Cariboni, D. Gatelli,  
M. Saisana, S. Tarantola

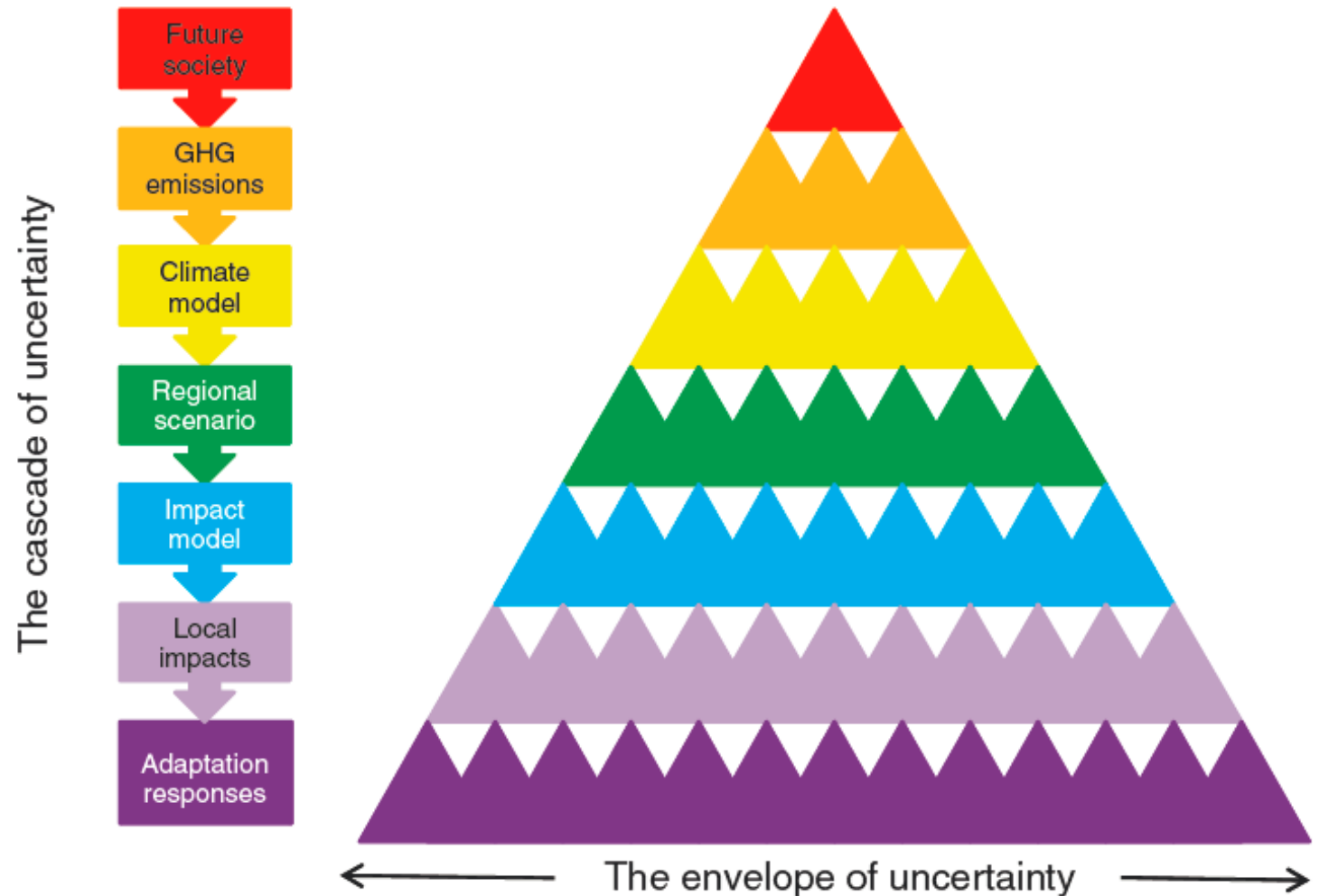
# GLOBAL SENSITIVITY ANALYSIS

The Primer

 WILEY



Uncertainty analysis:  
the study of the  
uncertainty in model  
output—see also  
uncertainty cascade



Source: <https://www.climate-lab-book.ac.uk/2014/cascade-of-uncertainty/>



Sensitivity analysis: the study of the relative importance of different input factors on the model output

## Sensitivity analysis can:

- surprise the analyst,
- uncover technical errors in the model,
- identify critical regions in the space of the inputs,



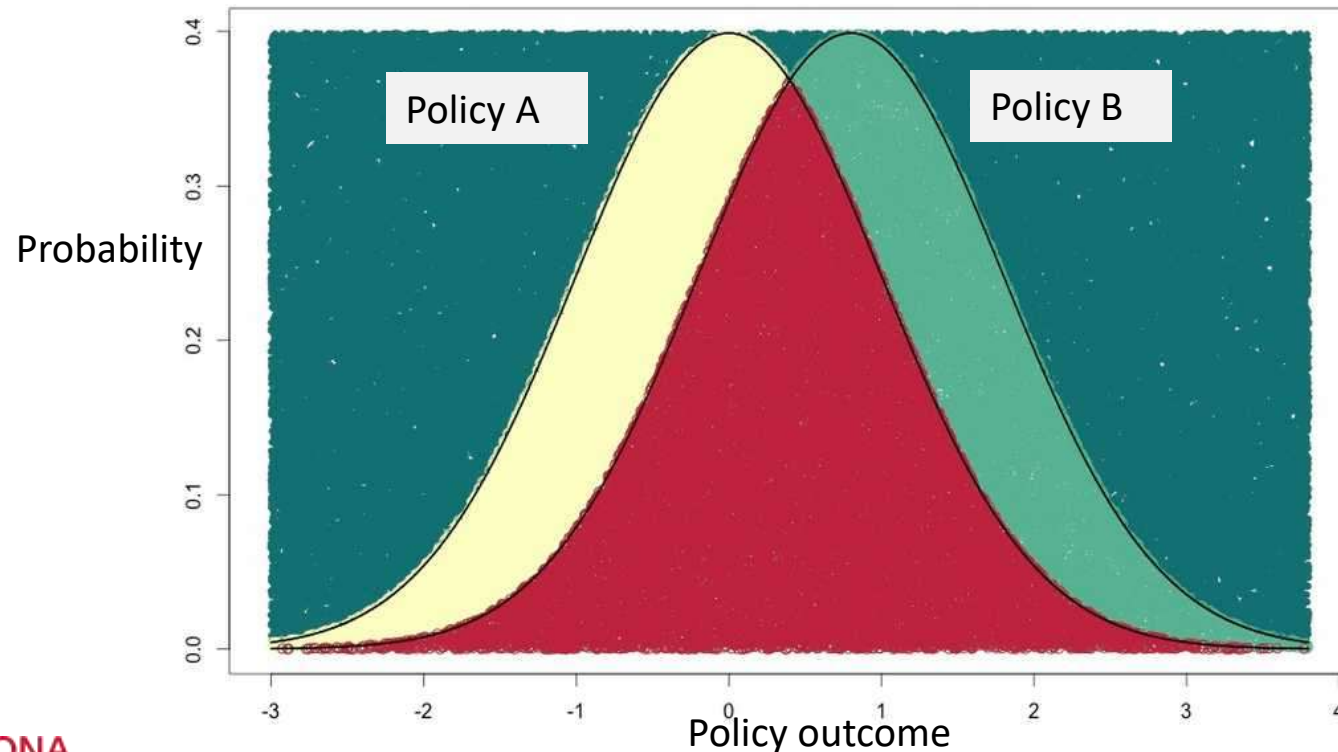
Source: The Simpson, 20th Television Animation  
(The Walt Disney Company)

## Sensitivity analysis can :

- surprise the analyst,
- uncover technical errors in the model,
- identify critical regions in the space of the inputs,
- establish priorities for research,
- simplify models
- falsify models (show that a model is false or irrelevant)
- defend against your own model being falsified

## Sensitivity analysis can:

verify whether policy options (or marketing strategies) can be distinguished from one another given the uncertainties in the system, ...



What method would one choose to perform sensitivity analysis?



Source: iStock by Getty images

What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

$$y = f(x_1, x_2, \dots, x_k)$$

$$\left. \frac{\partial y}{\partial x_i} \right|_{x_i = x_i^0} \longleftarrow \text{Local}$$



What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

$$y = f(x_1, x_2, \dots, x_k)$$

$$\frac{x_i^0}{y^0} \frac{\partial y}{\partial x_i} \bigg|_{x_i=x_i^0} \longleftarrow \text{Local}$$

What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

$$y = f(x_1, x_2, \dots, x_k)$$

$$\frac{\text{std}(x_i)}{\text{std}(y)} \left. \frac{\partial y}{\partial x_i} \right|_{x_i=x_i^0} \longleftarrow \text{Hybrid}$$

$$\left. \frac{\partial y}{\partial x_i} \right|_{x_i=x_i^0}$$



Relative effect on  $y$  of perturbing  $x_i$  around its nominal value

$$\left. \frac{x_i^0}{y^0} \frac{\partial y}{\partial x_i} \right|_{x_i=x_i^0}$$

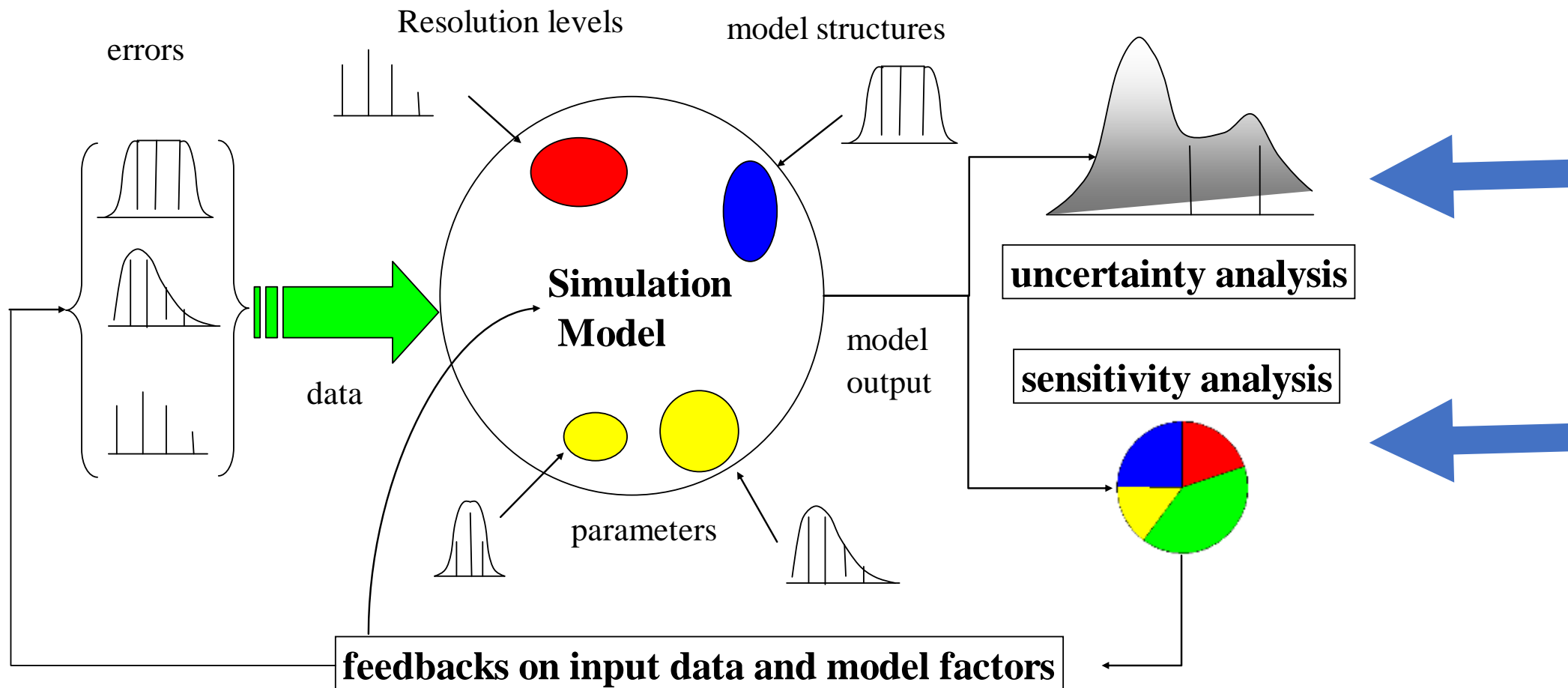


Relative effect on  $y$  of perturbing  $x_i$  by a fixed fraction of its nominal value

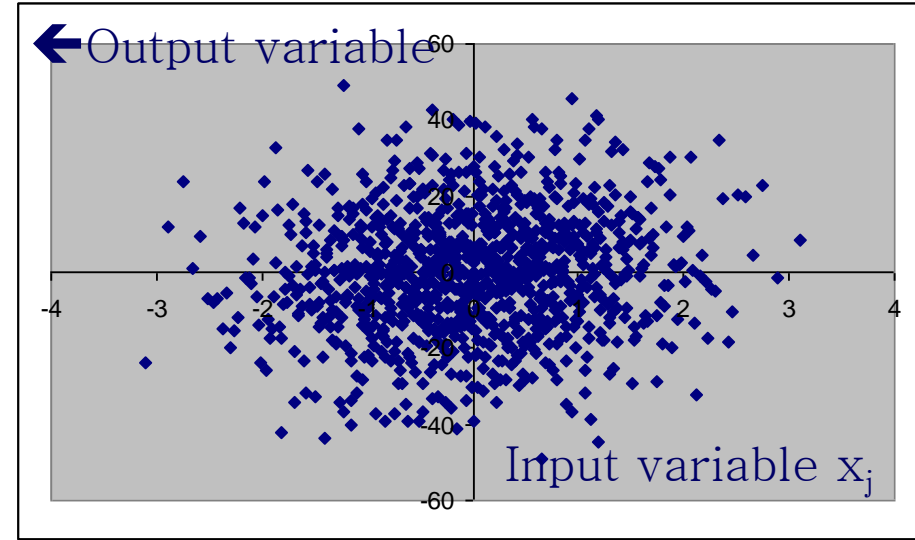
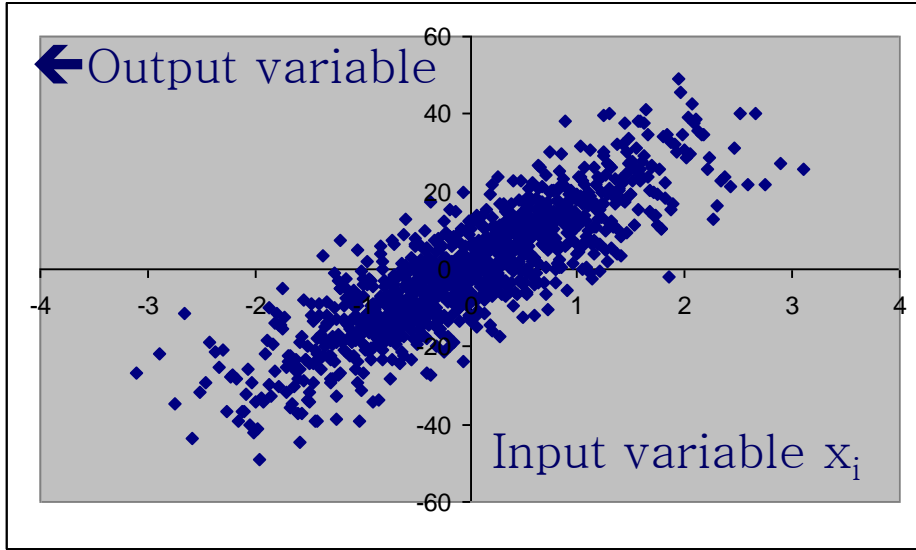
$$\left. \frac{std(x_i)}{std(y)} \frac{\partial y}{\partial x_i} \right|_{x_i=x_i^0}$$



Relative effect on  $y$  of perturbing  $x_i$  by a fixed fraction of its standard deviation



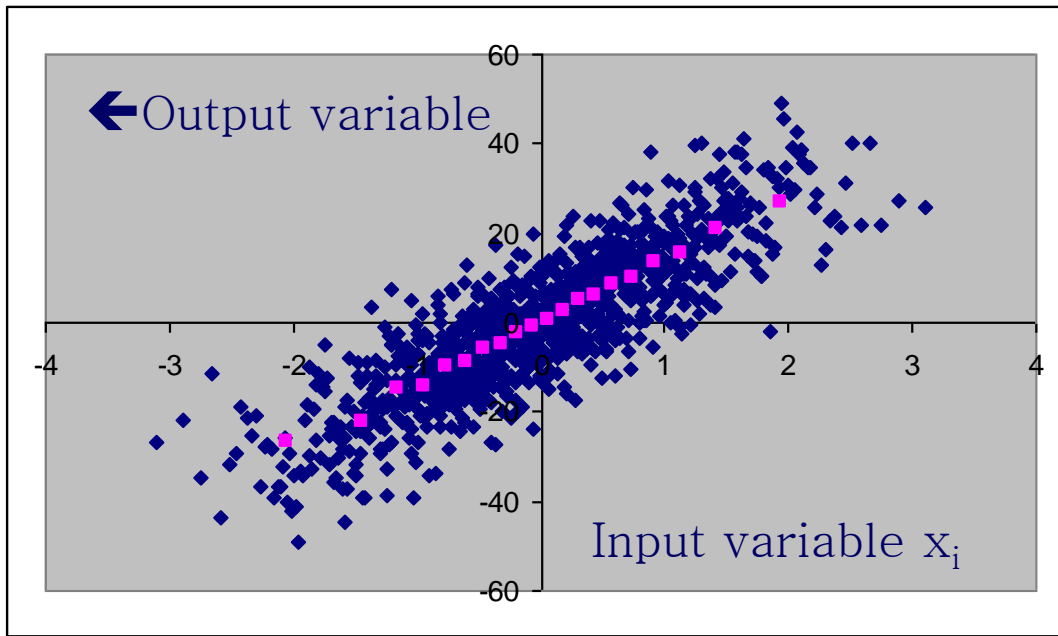
# An introduction to variance based methods



Plotting the output as a function of two different input factors

Which factor is more important?

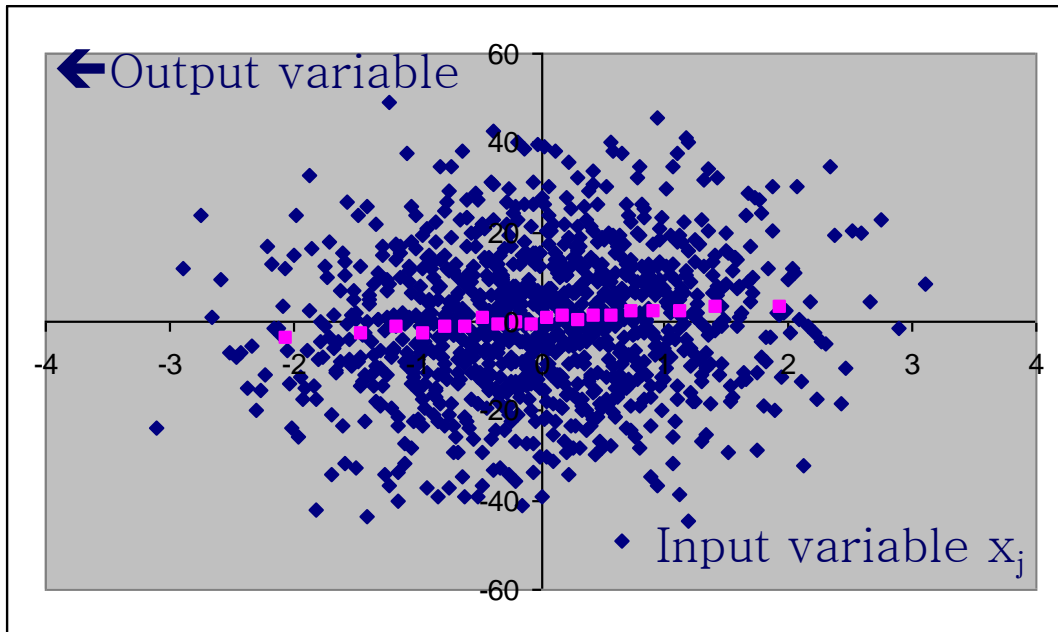


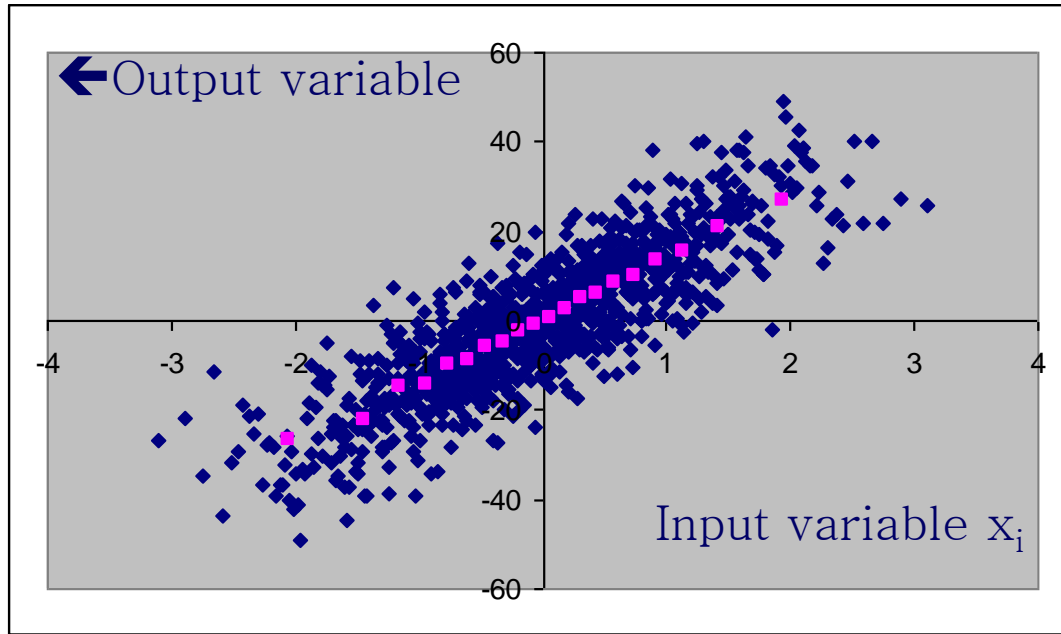


~1,000 blue points

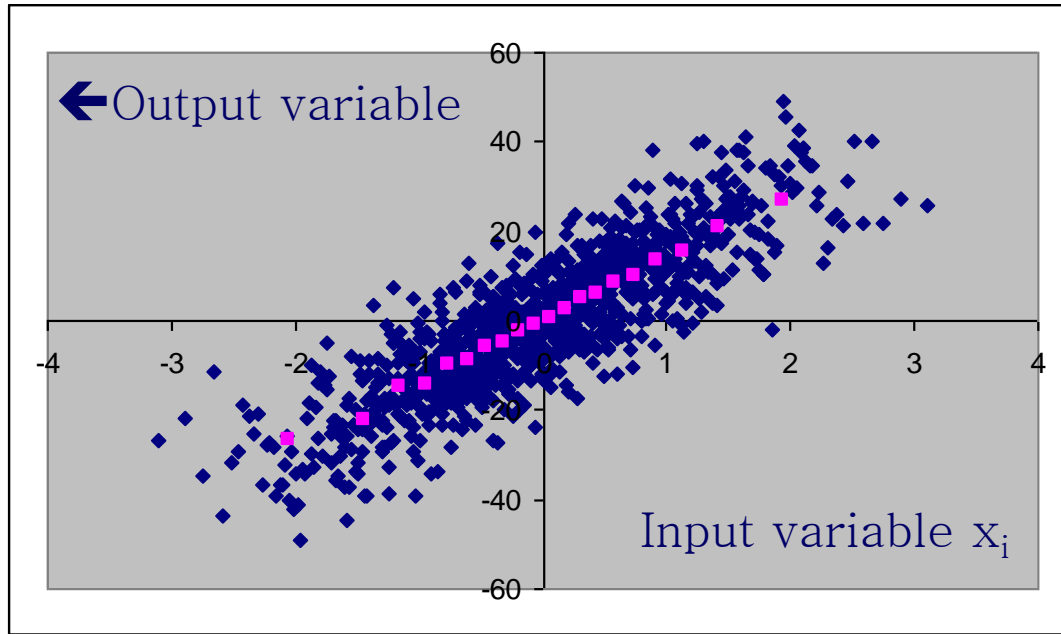
Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)



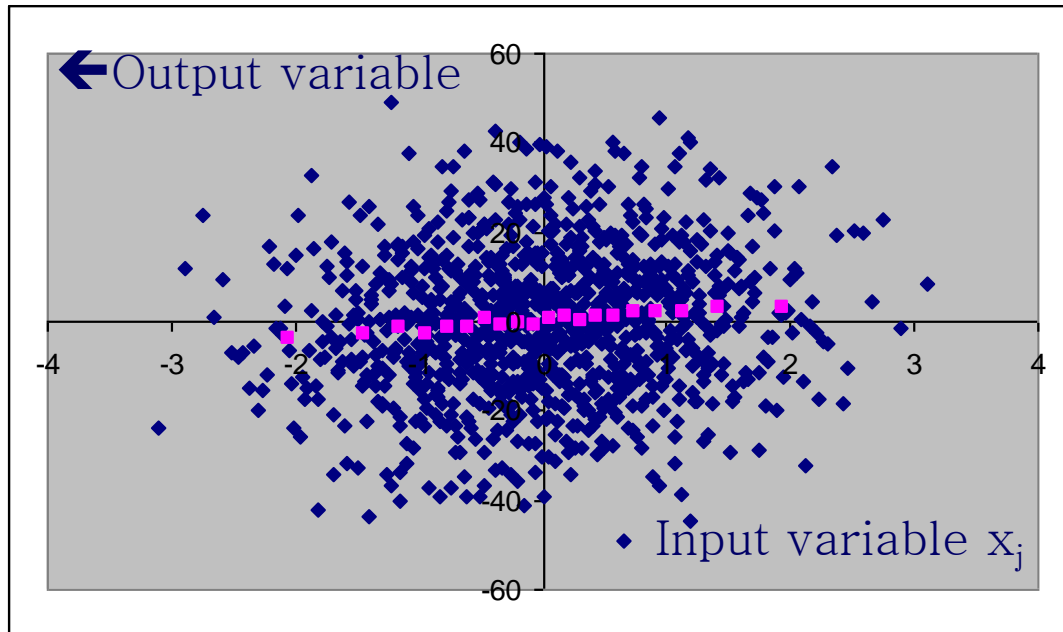
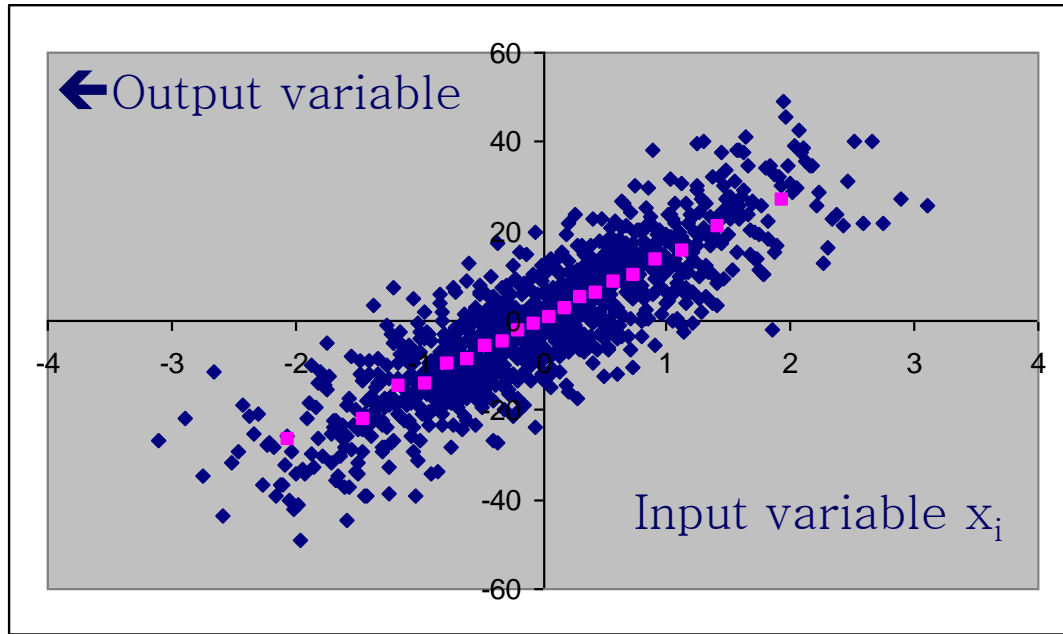


Each pink point is  $\sim E_{\mathbf{X}_{\sim i}}(Y|X_i)$



Taking the variance  
of the pink points one  
obtains a sensitivity  
measure

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$



Which factor  
has the highest  
 $V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$  ?

$$S_i = \frac{V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)}{V(Y)}$$

The partial variance divided by the total variance is the so-called sensitivity index of the first order

For additive models one can decompose the total variance as a sum of those partial variances

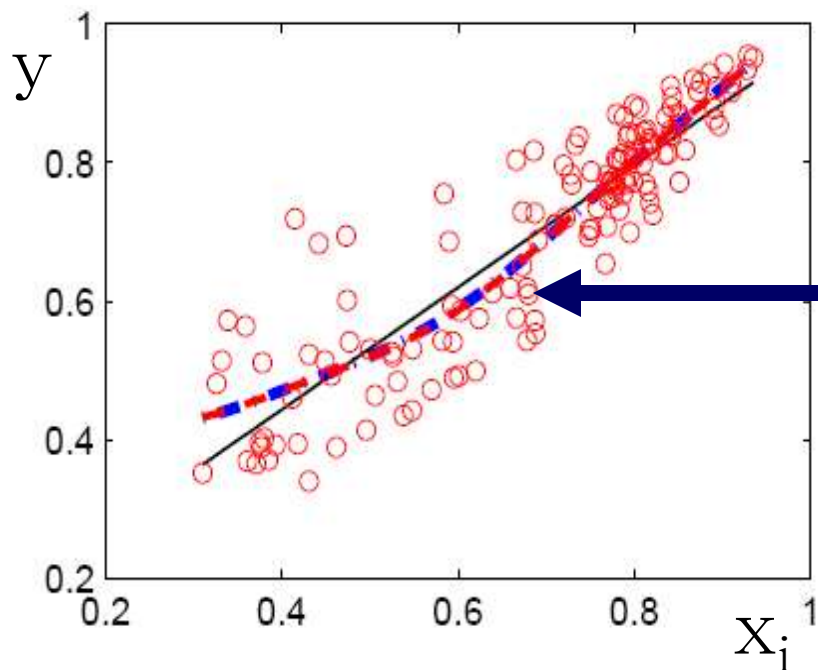
$$\sum_i V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) \approx V(Y)$$

... which is also how additive models are defined



$$S_i = \frac{V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)}{V(Y)}$$

The partial variance divided by the total variance is the so-called sensitivity index of the first order, identical in formulation to Pearson's correlation ratio



Smoothed curve:

$$\mathbf{E}_{\mathbf{x} \sim i} (y \mid x_i)$$

First order  
sensitivity index:

$$\frac{V_{x_i} (\mathbf{E}_{\mathbf{x} \sim i} (y \mid x_i))}{V(y)}$$

Pearson's correlation  
ratio

Smoothed curve

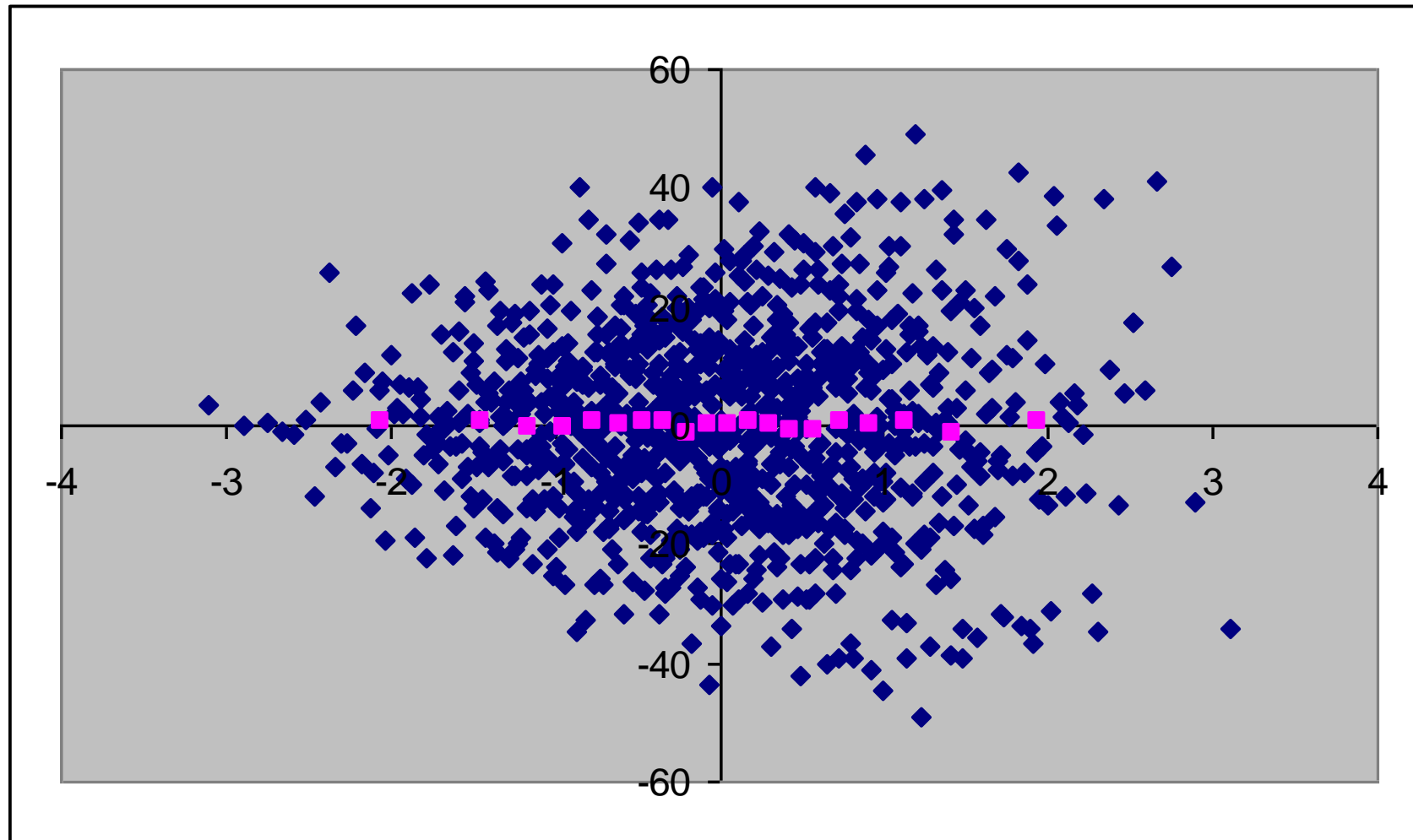
$$S_i \equiv \eta_i^2 := \frac{V_{x_i} (\mathbf{E}_{\mathbf{x}_{\sim i}} (y \mid x_i))}{V(y)}$$

First order sensitivity index

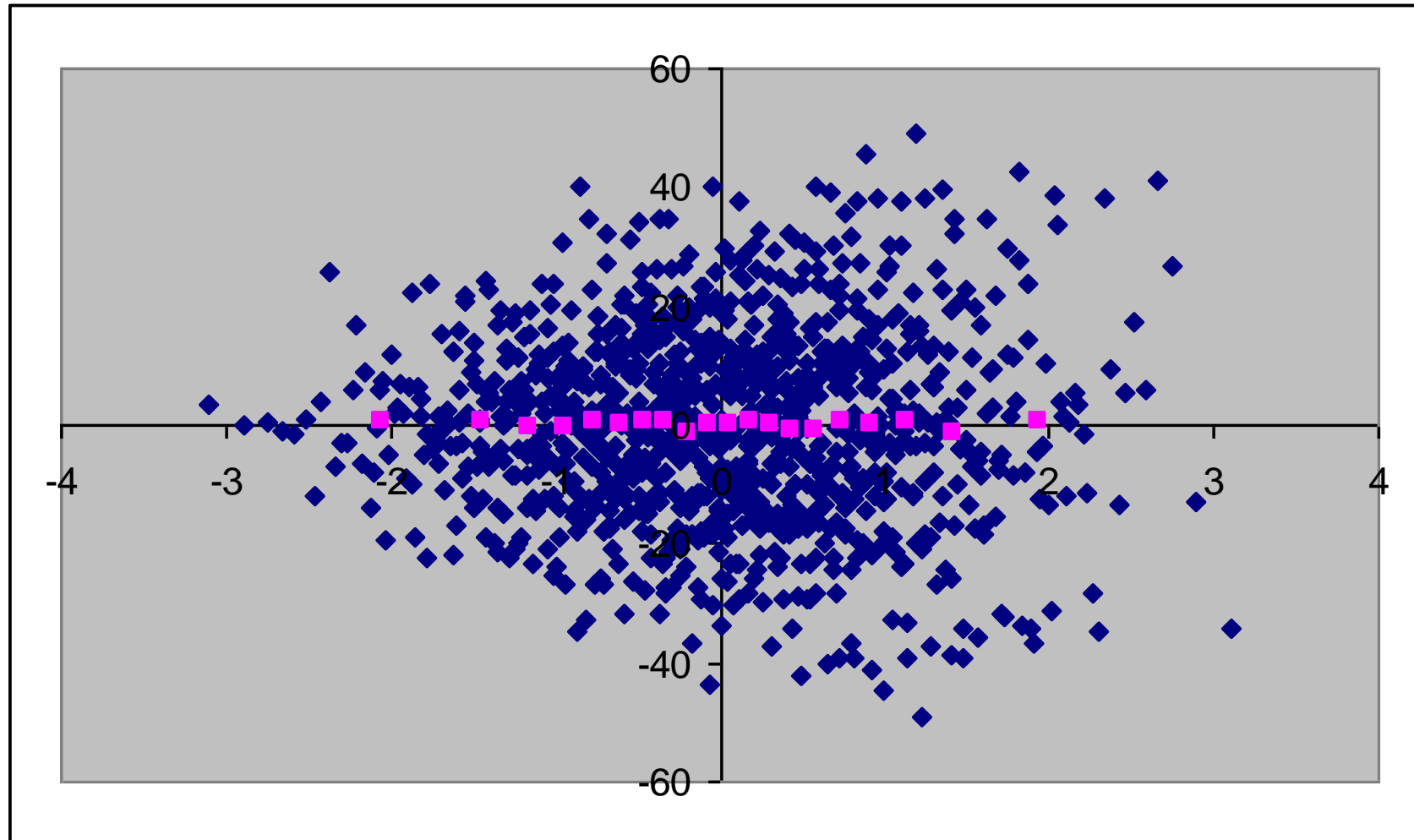
Unconditional  
variance

# Non additive models

Is  $S_i = 0$ ?



Is this factor non-important?





There are terms which capture  
two-way, three way,  $\dots$  interactions  
among variables

All these terms are linked by a  
formula

# Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_i V_i + \sum_{i,j>i} V_{ij} + \dots + V_{123\dots k}$$

# Variance decomposition (ANOVA)

The total variance can be decomposed into main effects and interaction effects up to the order  $k$ , the dimensionality of the problem (only for independent factors)

If fact interactions terms are awkward to handle: **just** the **second** order terms for a model with  $k$  factors are as many as  $k(k-1)/2 \dots$

(10 factors=45 second order terms)

How about a single 'importance' terms for all effects?

In fact such terms exist and can be computed easily, without knowledge of the individual interaction terms



Thus given a model  $f(X_1, X_2, \dots, X_3)$

Where the variance decomposition would

$$\text{read } \mathbf{1} = S_1 + S_2 + S_3 + S_{12} + S_{13} + S_{23} + S_{123}$$

We compute

$$T_1 = S_1 + S_{12} + S_{13} + S_{123}$$

$$T_2 = S_2 + S_{12} + S_{23} + S_{123}$$

$$T_3 = S_3 + S_{13} + S_{23} + S_{123}$$

# The measures and their 'settings' = when to use them



**Journal of the American Statistical Association** >

Volume 97, 2002 - Issue 459

Submit an article

Journal homepage

Enter keywords, authors, DOI, C

979

Views

286

CrossRef citations  
to date

6

Altmetric

Primary Article

## On the Relative Importance of Input Factors in Mathematical Models

*Safety Assessment for Nuclear Waste Disposal*

Andrea Saltelli & Stefano Tarantola

Pages 702-709 | Published online: 31 Dec 2011

Download citation

<https://doi.org/10.1198/016214502388618447>

The measures and their ‘settings’  
= when to use them

First order effect	Factor prioritization (orienting research)
Total effect	Factor fixing (model simplification)



# Computing the indices efficiently

## Making best use of model evaluations to compute sensitivity indices

Andrea Saltelli  

## Higher order Sobol' indices

Get access >

Art B. Owen , Josef Dick, Su Chen

*Information and Inference: A Journal of the IMA*, Volume 3, Issue 1, March 2014, Pages 59–81, <https://doi.org/10.1093/imaiai/iau001>

**Published:** 01 March 2014    **Article history** ▼

# Plenty of code available in R, MATLAB, and Python



<https://cran.r-project.org/web/packages/sensitivity/sensitivity.pdf>

<https://cran.rstudio.com/web/packages/sensobol/index.html>



<https://www.uqlab.com/> (in MatLab, by Bruno Sudret and his team)



SALib <https://salib.readthedocs.io/en/latest/>

...but there is more, in R, Python, Julia ...

# Advantages with variance based methods:

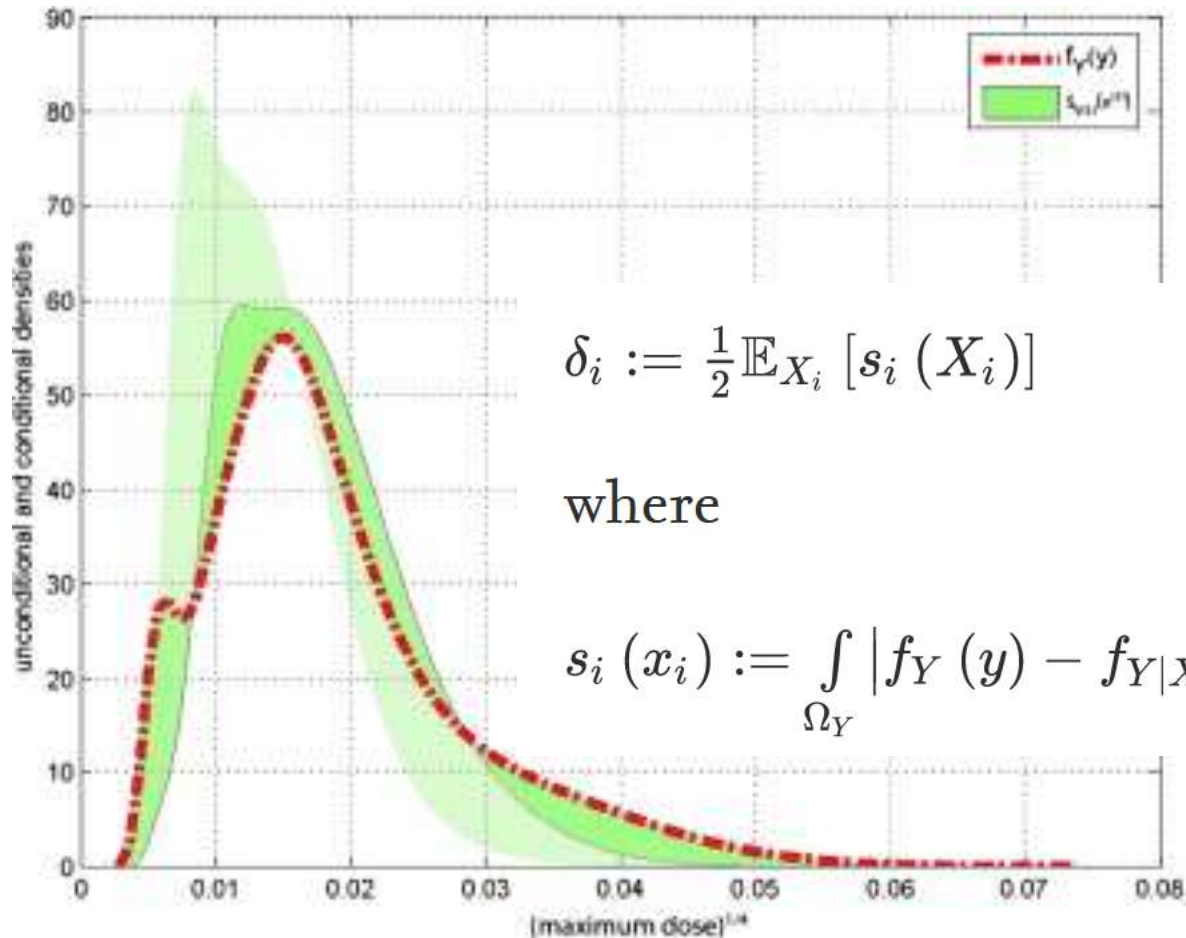
- graphic interpretation scatterplots
- statistical interpretation (ANOVA)
- expressed plain English
- working with sets
- relation to settings such as factor fixing and factor prioritization
- give the effective dimension



Chapter 1 and its  
exercises



... but there are other methods that can be used for different settings, e.g. moment independent methods, Shapley coefficients, reduced spaces, VARS ...



$$\delta_i := \frac{1}{2} \mathbb{E}_{X_i} [s_i (X_i)]$$

where

$$s_i (x_i) := \int_{\Omega_Y} |f_Y (y) - f_{Y|X_i=x_i} (y)| dy$$



Environmental Modelling & Software

Volume 34, June 2012, Pages 105-115



## Model emulation and moment-independent sensitivity analysis: An application to environmental modelling

E. Borgonovo <sup>a</sup>, W. Castaings <sup>b, c</sup>, S. Tarantola <sup>d</sup>  

Don't use One factor At a  
Time (OAT)

A geometric proof



Contents lists available at ScienceDirect

## Environmental Modelling & Software

journal homepage: [www.elsevier.com/locate/envsoft](http://www.elsevier.com/locate/envsoft)

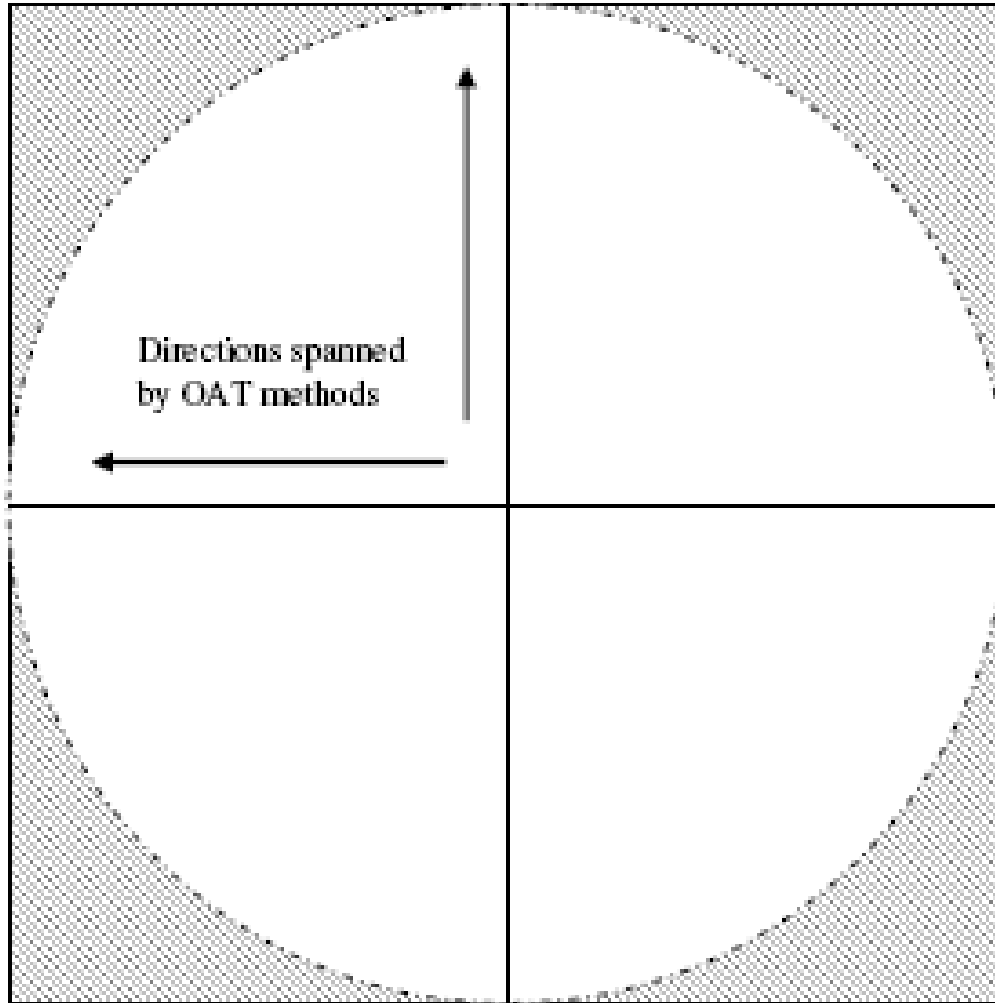


## How to avoid a **perfunctory** sensitivity analysis

Andrea Saltelli\*, Paola Annoni

*Joint Research Center, Institute for the Protection and Security of the Citizen, via E.Fermi, 2749, Ispra VA 21027, Italy*

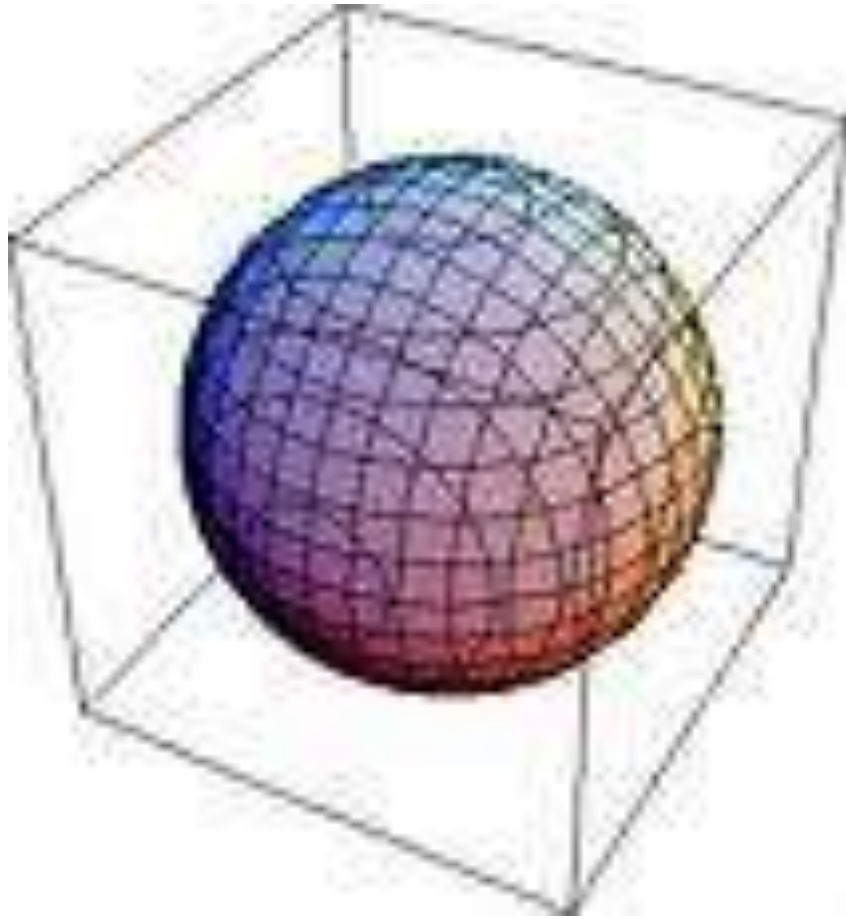
# OAT in 2 dimensions



Area circle  
/ area  
square = ?

$\sim 3/4$

# OAT in 3 dimensions



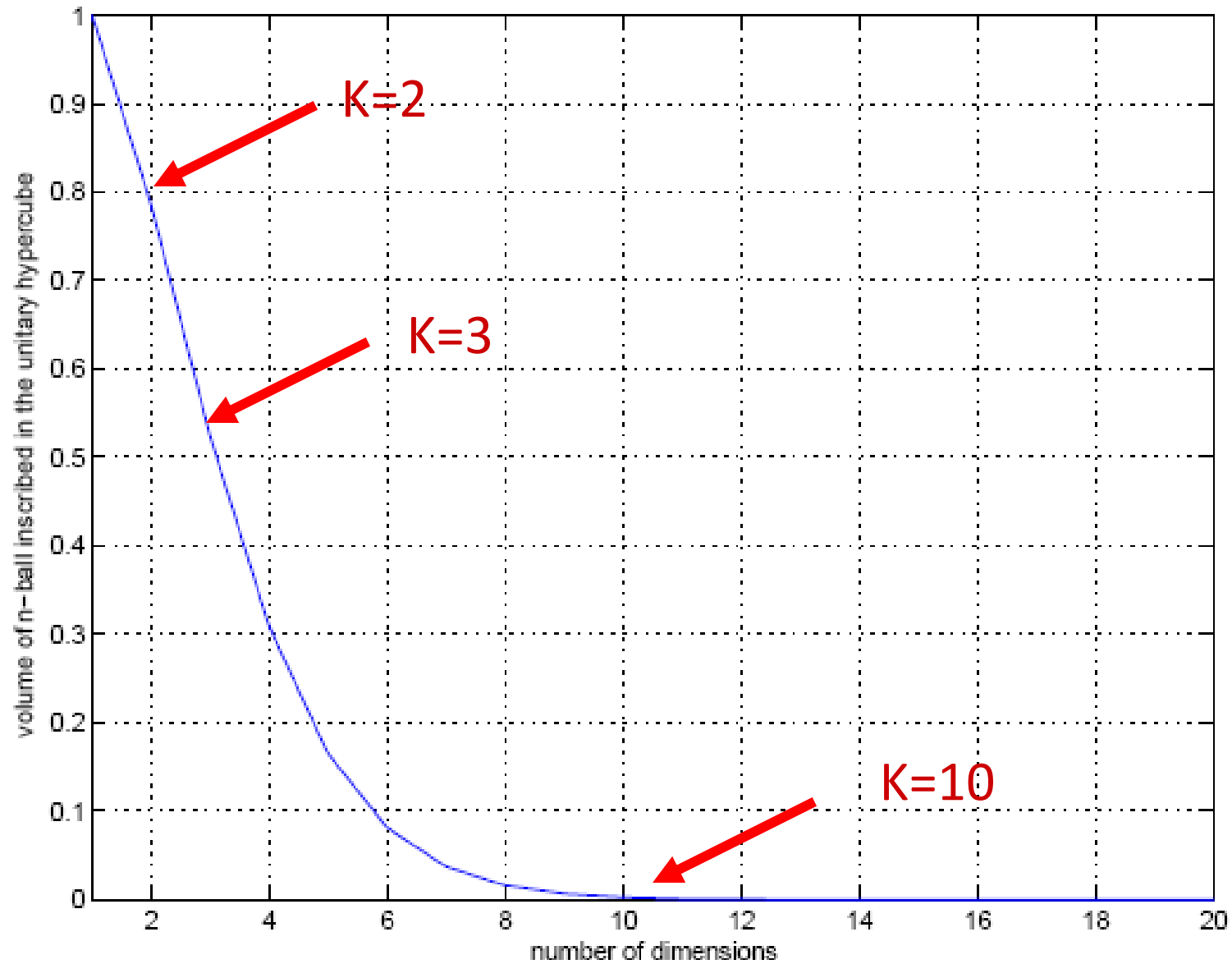
Volume sphere /  
volume cube = ?

$\sim 1/2$

OAT in 10 dimensions; Volume  
hypersphere / volume ten dimensional  
hypercube =?  $\sim 0.0025$



# OAT in k dimensions



OAT does not capture interactions

➔ The resulting analysis is non conservative



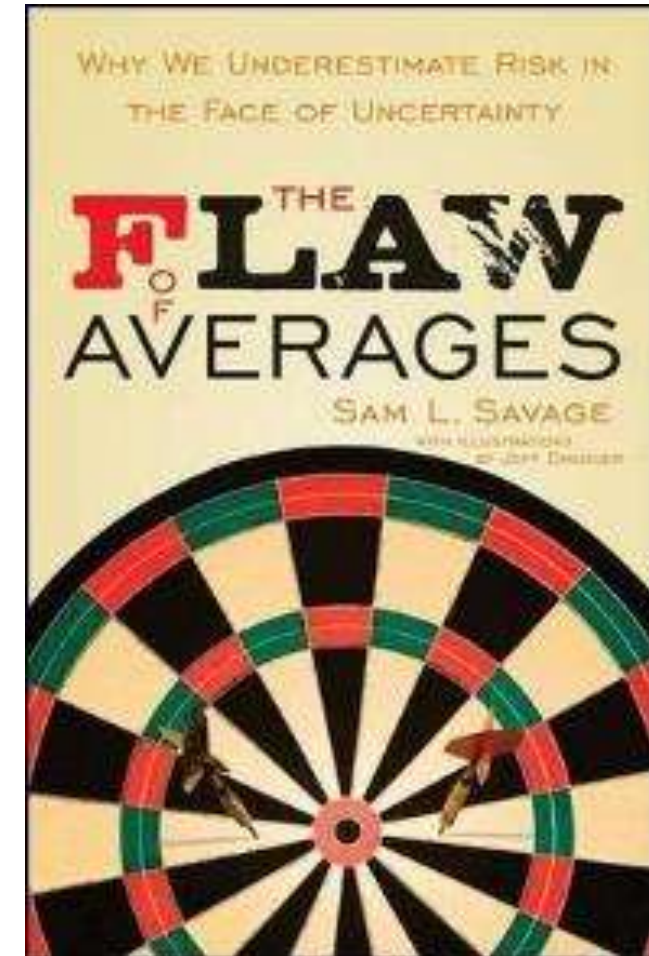
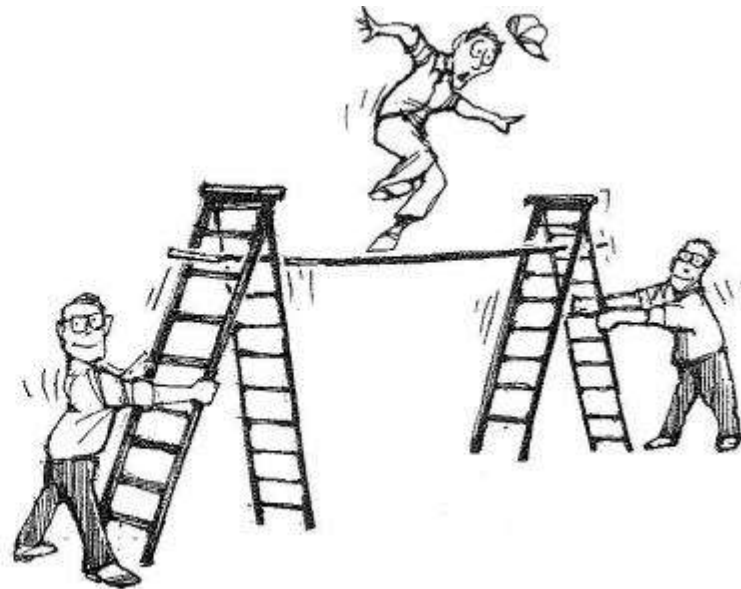
# How would you test the scaffolding?

How coupled ladders are shaken in most of available literature



≠

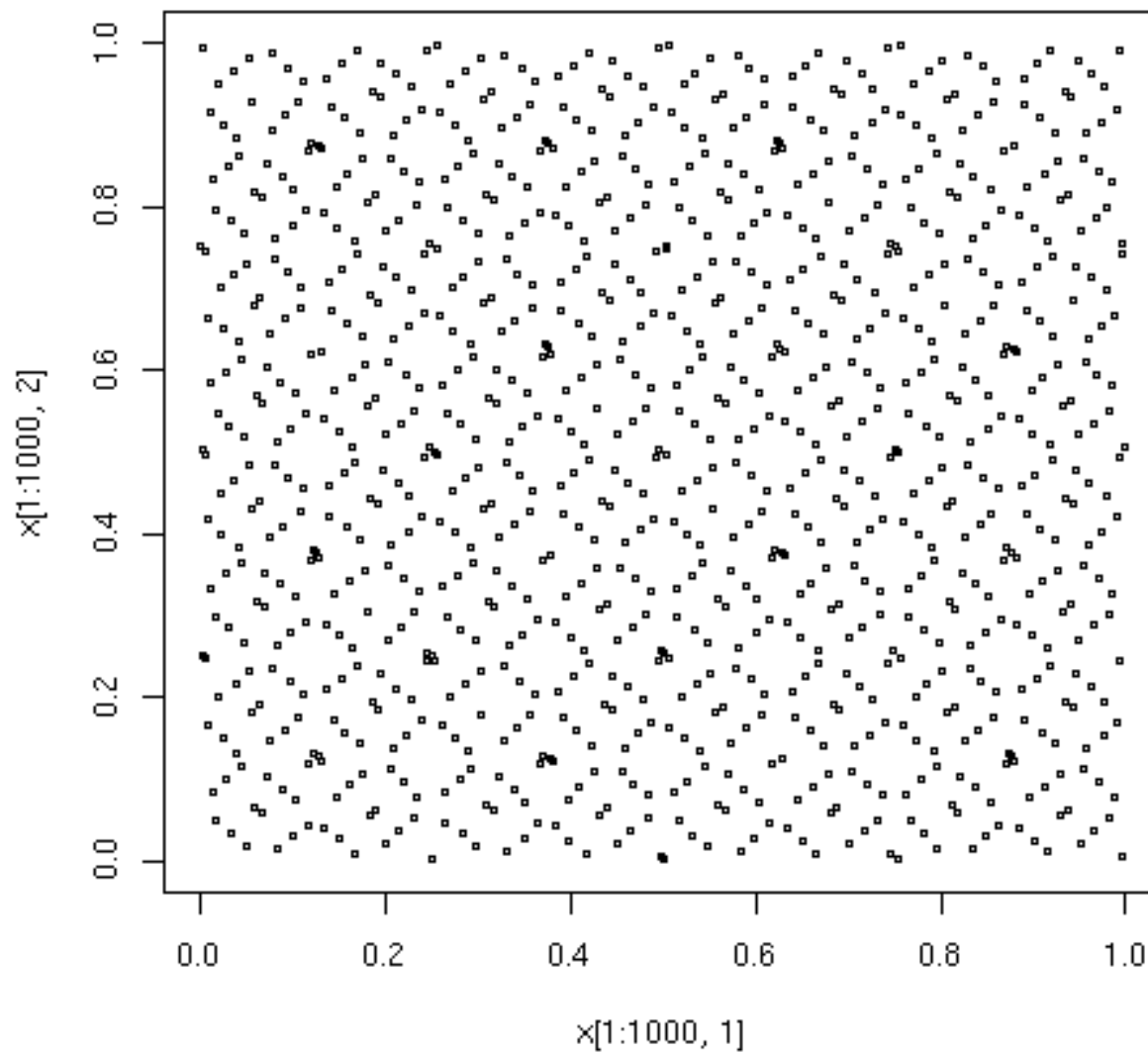
How to shake coupled ladders



# Quasi random sequences



Ilya M. Sobol'

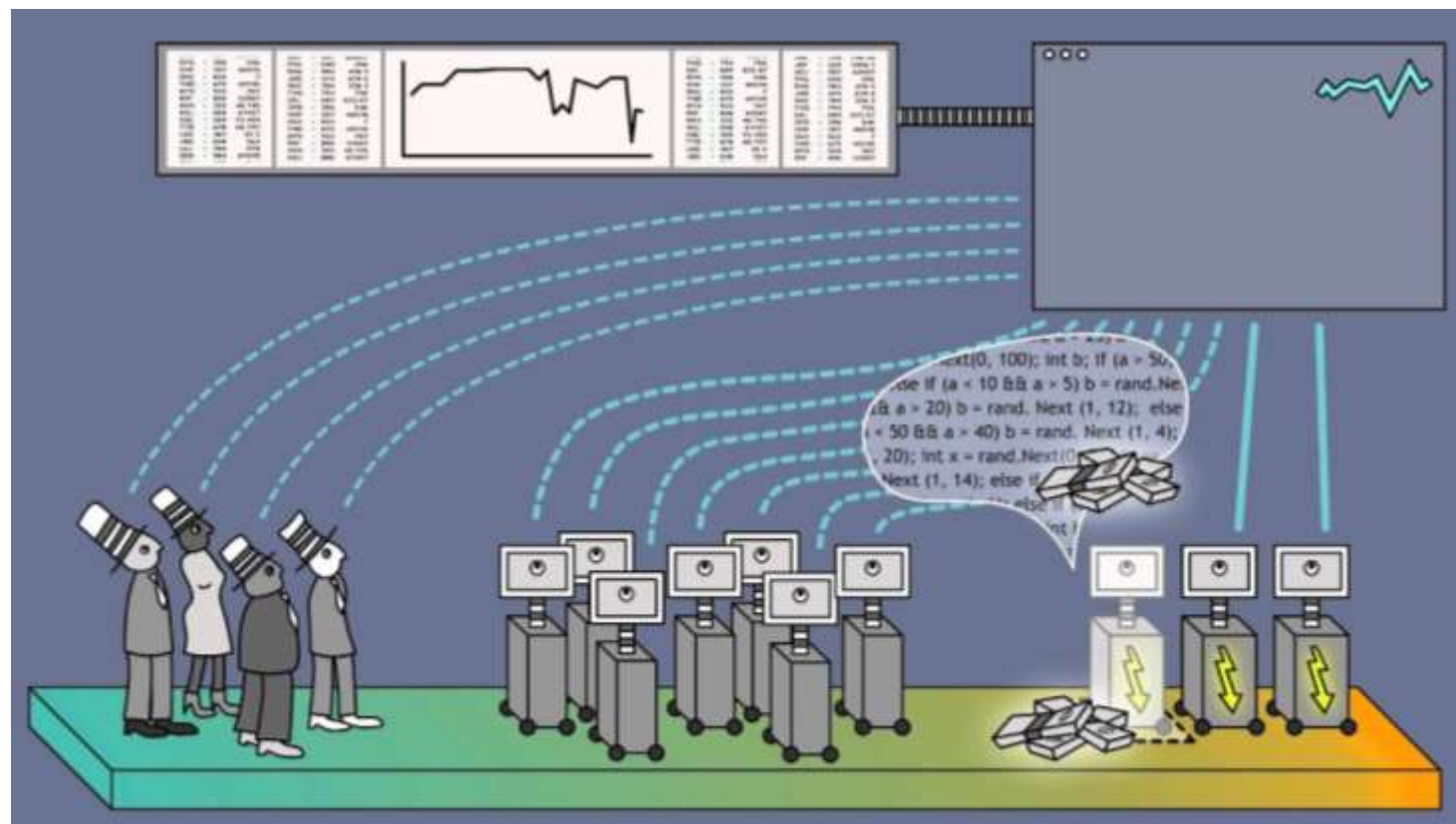


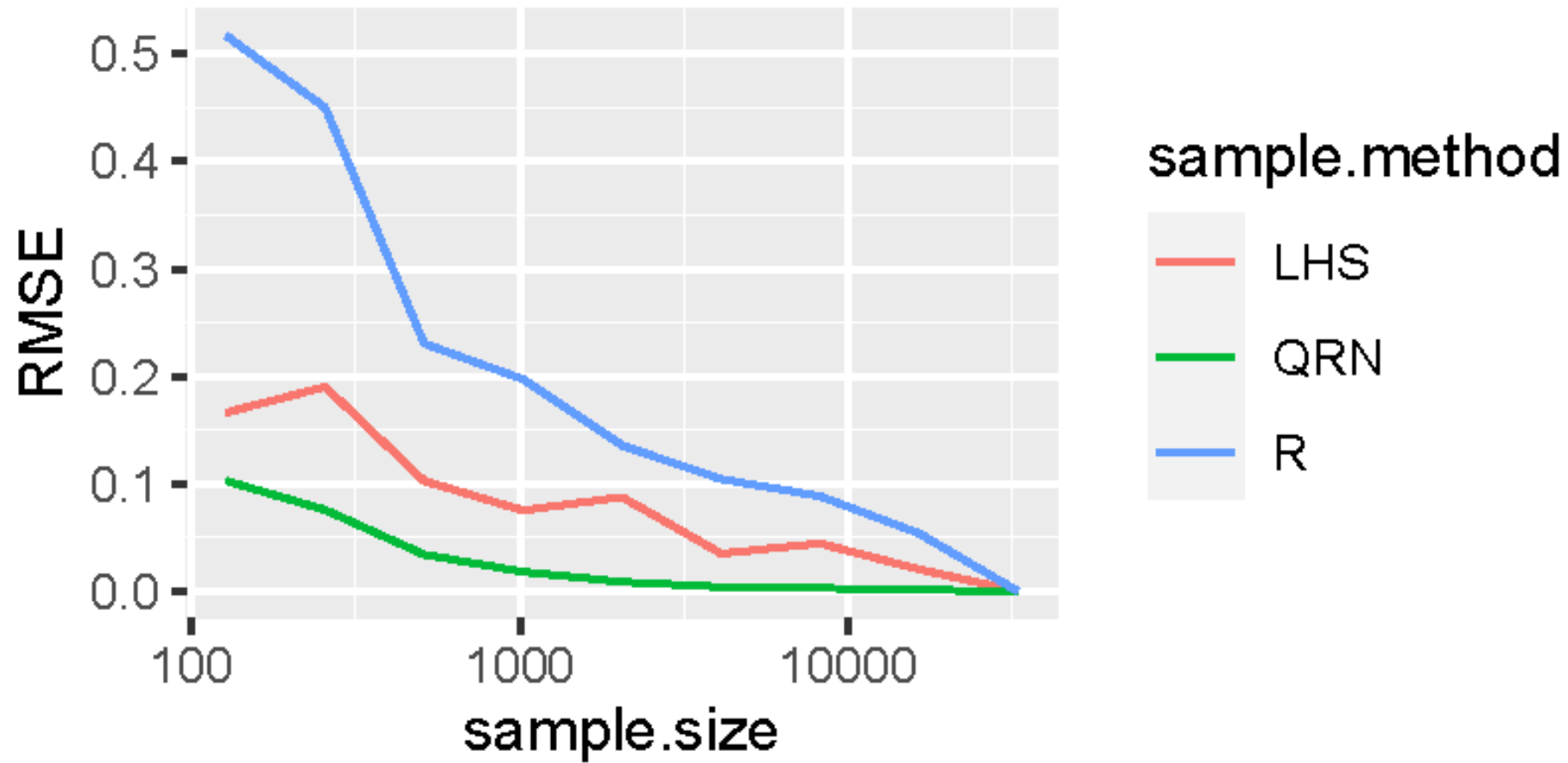
[Submitted on 10 May 2015]

# Exploring multi-dimensional spaces: a Comparison of Latin Hypercube and Quasi Monte Carlo Sampling Techniques

Sergei Kucherenko, Daniel Albrecht, Andrea Saltelli

Sobol' LP-TAU  
are used in high  
frequency trading





Root mean square error with different designs

Don't run the model just once

There is much to learn by running the model a few times, especially during model building



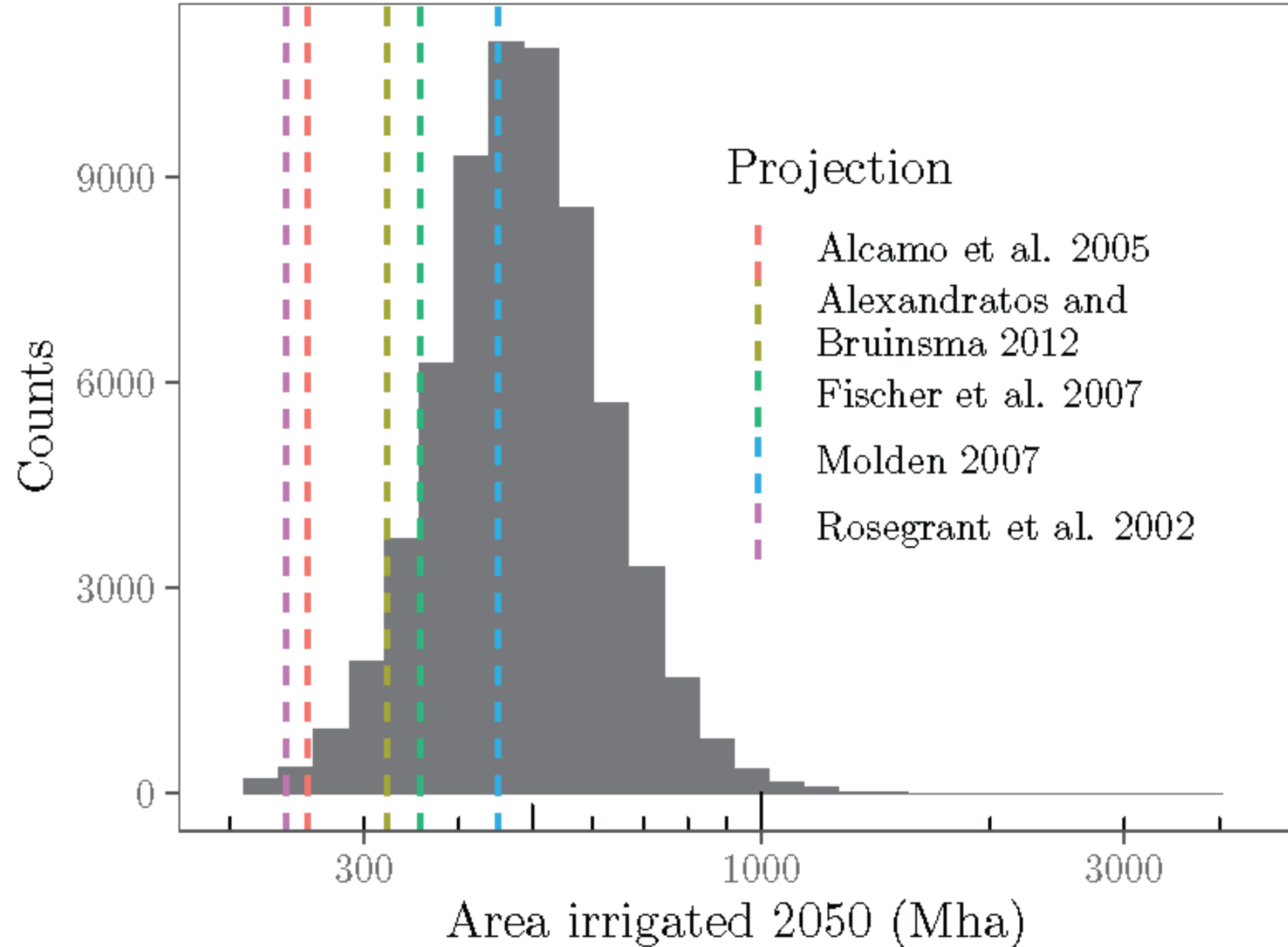
Lubarsky's Law of Cybernetic Entomology:  
there is always one more bug!



Model routinely used to produce point estimates may become non-conservative when the uncertainty is plugged in

# Current Models Underestimate Future Irrigated Areas

- How much land will need to be irrigated by the year 2050?
- Here the dashed lines represent deterministic model predictions from different models and datasets (from FAO & others organizations);
- An uncertainty analysis (grey histogram) reveals that the models are non-conservative: the need might be much larger





Don't sample just parameters and  
boundary conditions

Explore thoroughly the space of the  
assumptions

One can sample more than just factors:

- modelling assumptions,
- alternative data sets,
- resolution levels,
- scenarios ...

---

## Assumption

## Alternatives

---

Number of indicators

- all six indicators included or one-at-time excluded (6 options)

---

Weighting method

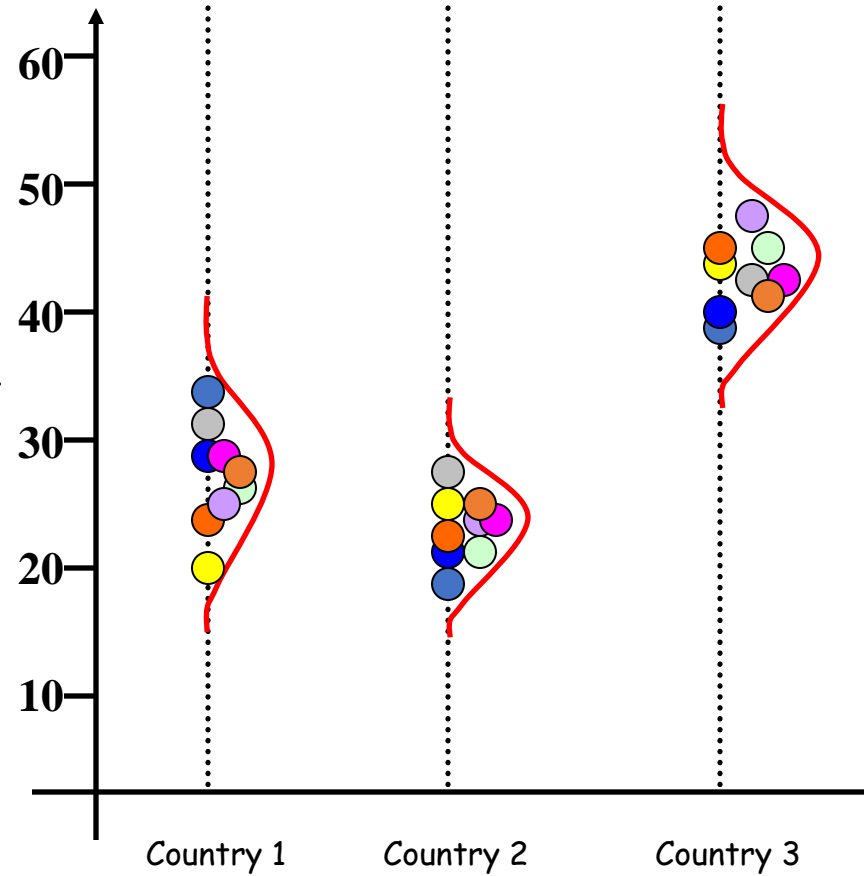
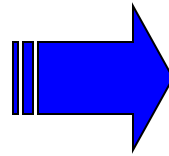
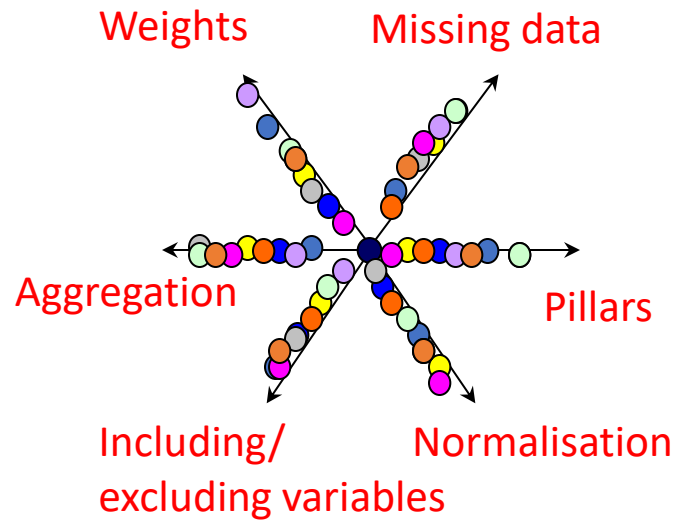
- original set of weights,
- factor analysis,
- equal weighting,
- data envelopment analysis

---

Aggregation rule

- additive,
  - multiplicative,
  - Borda multi-criterion
-

# Space of alternatives

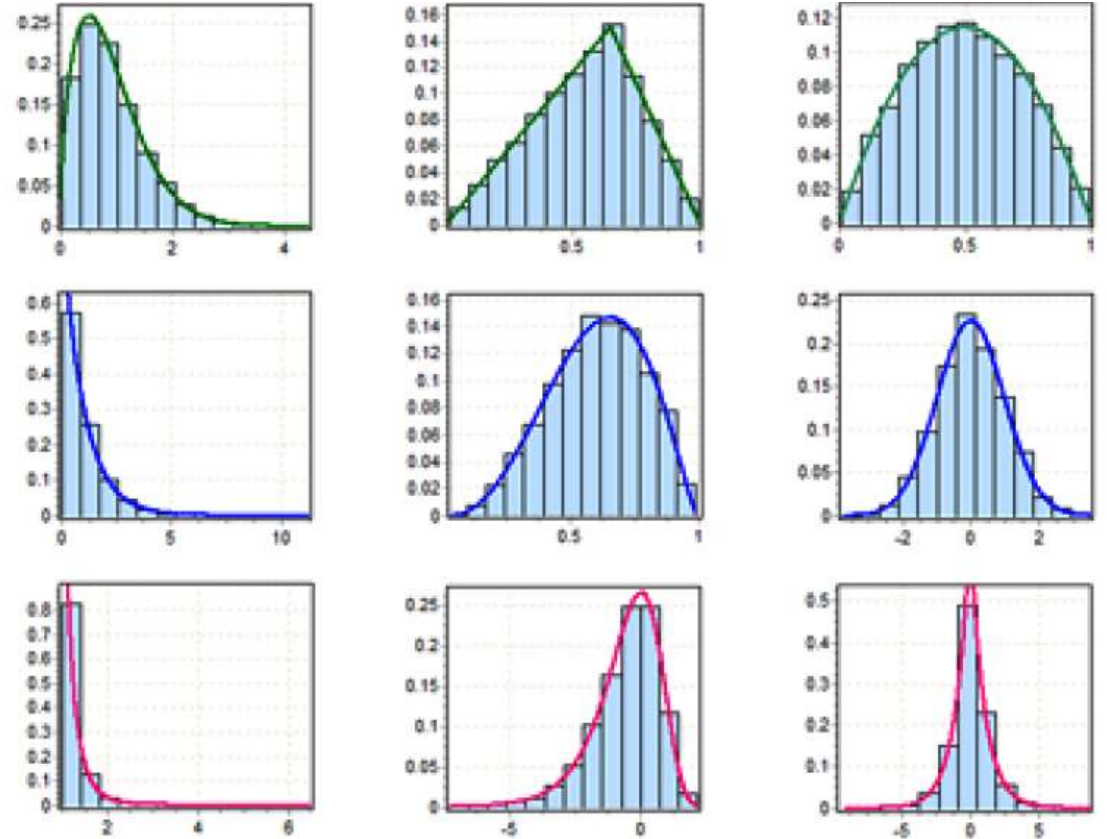


# Building a Monte Carlo analysis

$$\begin{array}{ccc} x_{11} & x_{12}\dots & x_{1k} \\ x_{21} & x_{22}\dots & x_{2k} \\ \dots & \dots & \dots \\ x_{N1} & x_{N2} & x_{Nk1} \end{array}$$

Input matrix: each column is a sample of size  $N$  from the distribution of a factor

Each row is a sample trial of size  $k$  to generate a value of  $y$



Examples of distributions of input factors

NEVER vary all factors of the same amount

Be it 5%, 10%, or 20%



# New WHO estimates: Up to 190 000 people could die of COVID-19 in Africa if not controlled

07 May 2020

**Brazzaville** – Eighty-three thousand to 190 000 people in Africa could die of COVID-19 and 29 million to 44 million could get infected in the first year of the pandemic if containment measures fail, a new study by the World Health Organization (WHO) Regional Office for Africa finds. The research, which is based on prediction modelling, looks at 47 countries in the



Speculative scenario in which ten uncertain input probabilities are increased by an arbitrary 10% — as if they were truly equally uncertain — with no theoretical or empirical basis for such a choice





In a numerical experiment relating to a real-life application the range of uncertainty of each input is crucial input to the analysis, and often the most expensive to get

Why is all this important? Fishing expeditions and forking paths ...





Jorge Luis Borges  
(1899–1986)



Taking different  
narratives within the  
same novel like Ts'ui Pên

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no “fishing expedition” or “p-hacking” and the research hypothesis was posited ahead of time\*

Andrew Gelman<sup>†</sup> and Eric Loken<sup>‡</sup>

14 Nov 2013

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no “fishing expedition” or “p-hacking” and the research hypothesis was posited ahead of time\*

Andrew Gelman<sup>†</sup> and Eric Loken<sup>‡</sup>

14 Nov 2013

Why this matters?



**PNAS**

RESEARCH ARTICLE

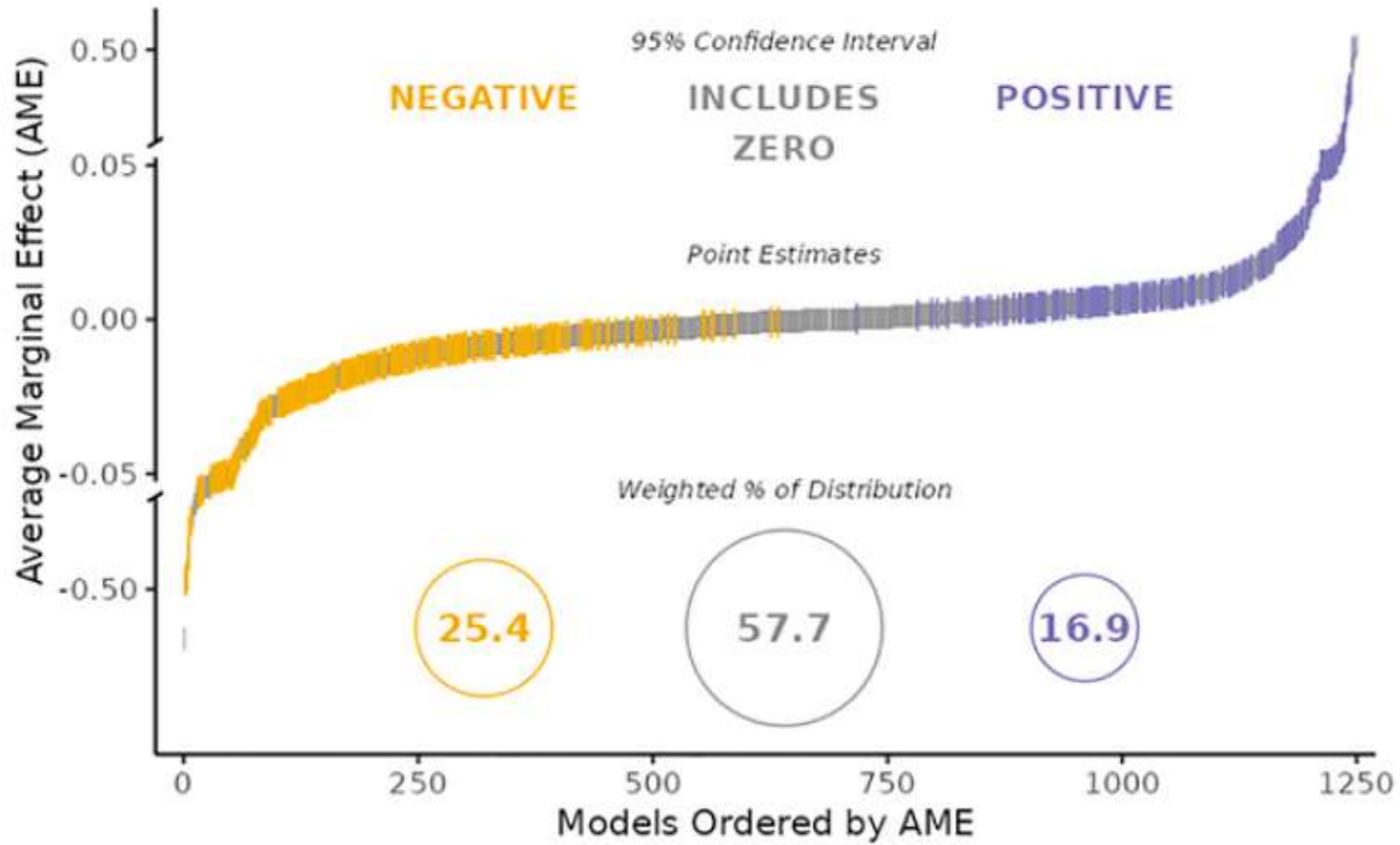
SOCIAL SCIENCES

 OPEN ACCESS

**Observing many researchers using the same data and hypothesis reveals a hidden universe of uncertainty**

Edited by Douglas Massey, Princeton University, Princeton, NJ; received March 6, 2022; accepted August 22, 2022





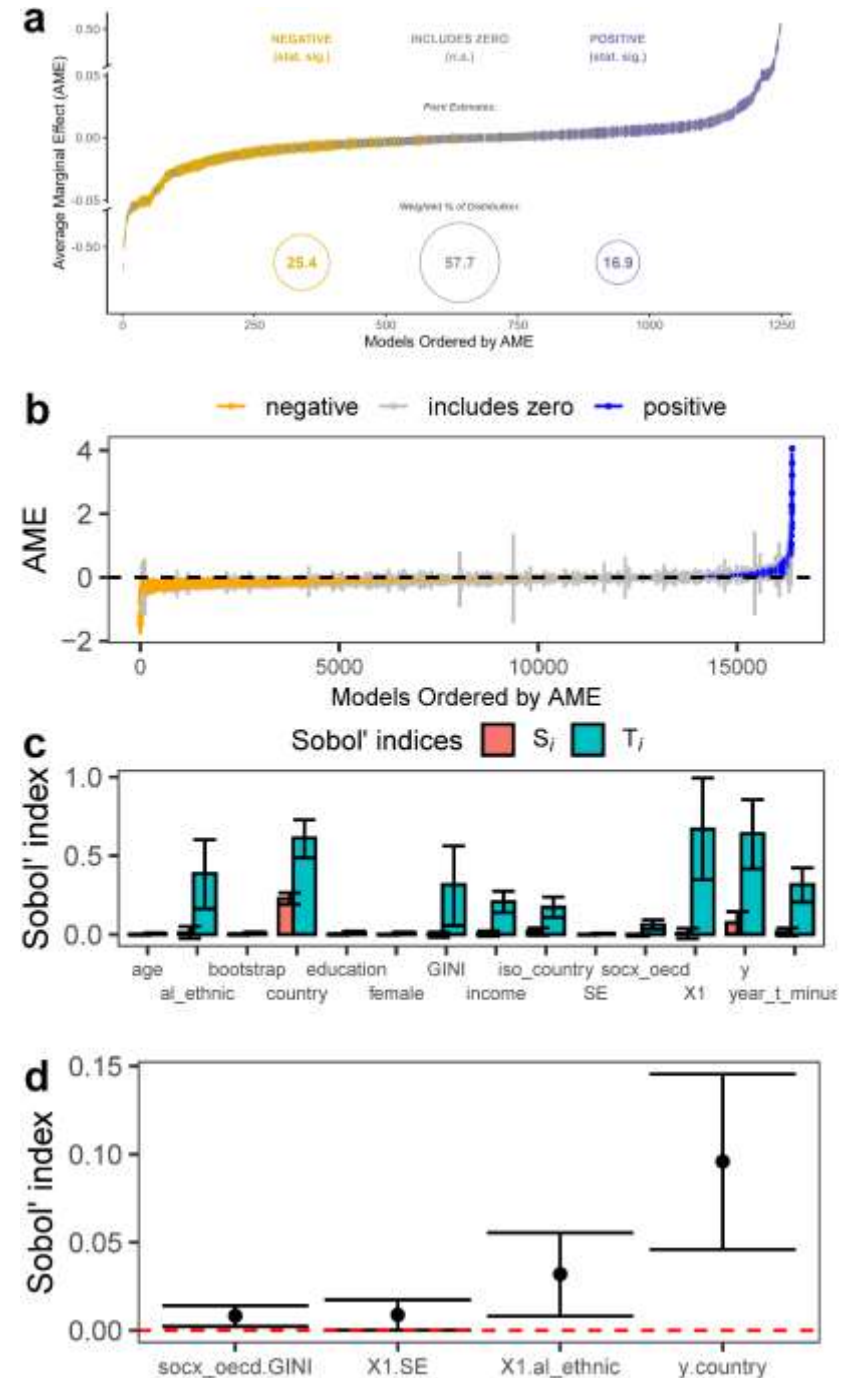
“Will different researchers [73 teams] converge on similar findings when analyzing the same data?”

…teams’ results varied greatly, ranging from large negative to large positive effects”  
(Massey et al. 2022)

Global sensitivity analysis can chart the garden before you enter ...



Andrea Saltelli, Arnald Puy, Alessio Lachi, and Nate Breznau, 2024, Global sensitivity analysis unveils the hidden universe of uncertainty in multiverse studies, MetaArXiv Preprints, <https://osf.io/preprints/metaarxiv/b67w9>.

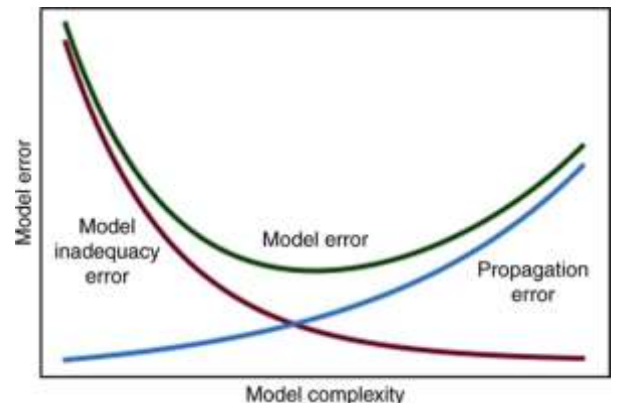


Sensitivity analysis can assist sociology of quantification in activities of de- and re-construction (e.g. for statactivists)



Modelling of the modelling process, to

- ➔ retrace what was assumed
- ➔ check the level of complexity
- ...



...

→ check simultaneously technical and normative quality

Example use SA to ascertain that an algorithm does not make implicit use of protected attributes

### More reading for the Machine Learning students

Vannucci, G., Siciliano, R., & Saltelli, A. (2024). Enhancing Variable Importance in Random Forests: A Novel Application of Global Sensitivity Analysis (arXiv:2407.14194). arXiv.  
<http://arxiv.org/abs/2407.14194>.

Bénesse, C., Gamboa, F., Loubes, J.-M., & Boissin, T. (2022). Fairness seen as Global Sensitivity Analysis. *Machine Learning*.  
<https://doi.org/10.1007/s10994-022-06202-y>

#### PROTECTED ATTRIBUTES:

- Age
- Disability
- National Origin
- Race/color
- Religion
- Sex
- (From the US Equal Opportunity Employment Commission)





Close to multiverse analysis

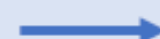
Close to NUSAP



Global sensitivity analysis



Modelling of the modelling process



Sensitivity auditing

Parametric uncertainty

Model uncertainty

Uncertainty in the modelling process

Political and normative dimensions

Sensitivity auditing : “Sensitivity auditing is a wider consideration of the effect of all types of uncertainty, including structural assumptions embedded in the model, and subjective decisions taken in the framing of the problem” (European Commission, 2021).



Better Regulation  
**TOOLBOX**

November 2021

European Commission. November 2021. “Better Regulation: Guidelines and Toolbox.”

[https://ec.europa.eu/info/law/law-making-process/planning-and-proposing-law/better-regulation-why-and-how/better-regulation-guidelines-and-toolbox\\_en](https://ec.europa.eu/info/law/law-making-process/planning-and-proposing-law/better-regulation-why-and-how/better-regulation-guidelines-and-toolbox_en)

---

## **What do I make of your latinorum? Sensitivity auditing of mathematical modelling**

---

Saltelli, A., Guimarães Pereira, Â., Van der Sluijs, J.P. and Funtowicz, S.



# The End



<https://mstdn.social/@AndreaSaltelli/>